Mapping the hydrodynamics response to initial conditions

Rapidity dependence of elliptic flow versus eccentricity

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In hydro calculations

$$T^{\mu\nu}\Big|_{initial} \rightarrow \frac{d^3N}{dp^3}\Big|_{final}$$

Why to study this relation

The relation between elliptic flow (V_2) and the eccentricity (ϵ_2) has been studied at midrapidity having a linear dependence to each other

$$V_2 = k \cdot \epsilon_2$$

The goal of this study is to understand how this relation can be extended for different values of rapidity. So maybe

$$V_2(Y) = k \cdot \epsilon_2(\eta = Y)?$$

Initial Conditions

The energy-momentum tensor has all the information we need for the initial condition of the hydrodynamic system.

We have the energy density:

$$T^{00} = \frac{d^3 E}{dx^3}$$

MUSIC code

- Initial conditions provided by the user
- Parameters
- Evolution of the system until freeze-out
- Distribution of number of particles per momentum

Final Observables

We have the number of particles distribution

We can calculate the "flows"

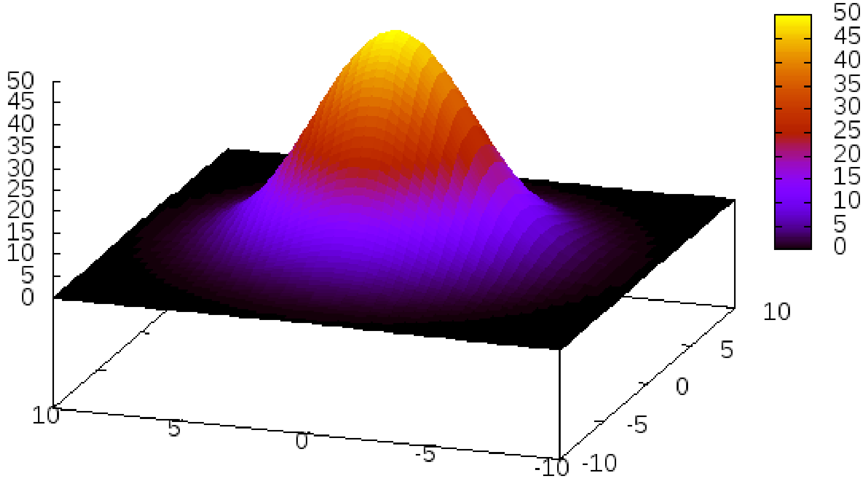
We do it with a Fourier series:

$$E\frac{dN}{dp^3}=\frac{dN}{p_T\,dp_T\,d\varphi\,dY}=\sum_{n=-\infty}^{\infty}V_n e^{-in(\varphi-\phi_n)}$$

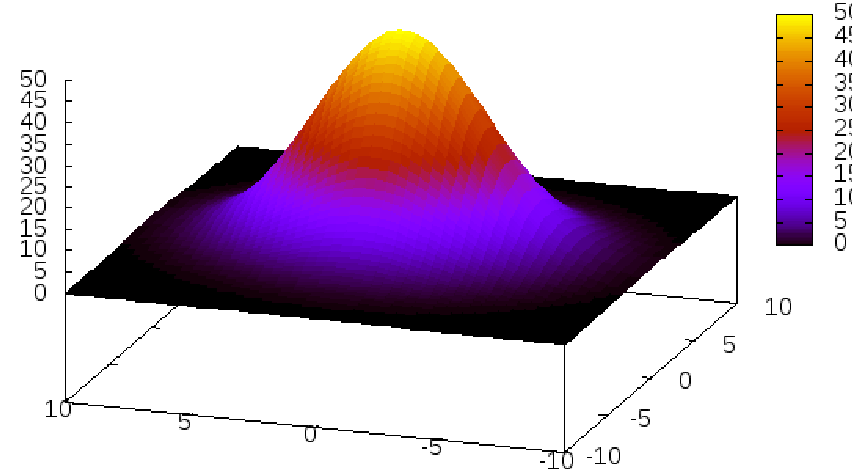
where V_2 is called the elliptic flow and $V_2=V_2(Y)$

$$T^{00} = A \cdot \exp\left(\frac{-r^2\left(1+\epsilon_2 \cdot \cos(2\varphi)\right)}{2 \cdot B^2}\right)$$

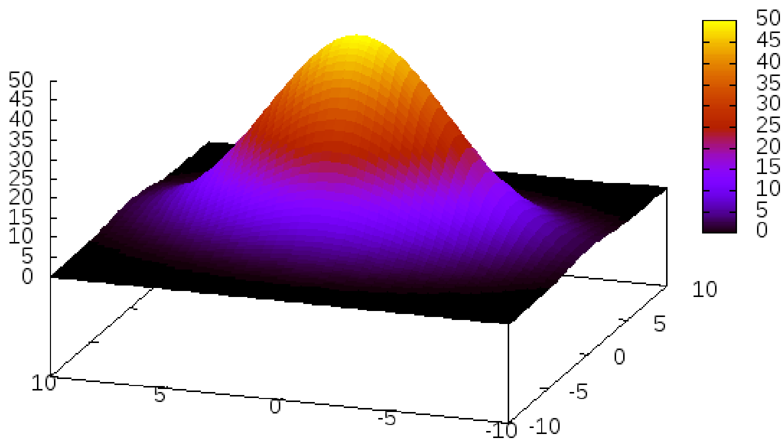
$\epsilon_2 = 0$



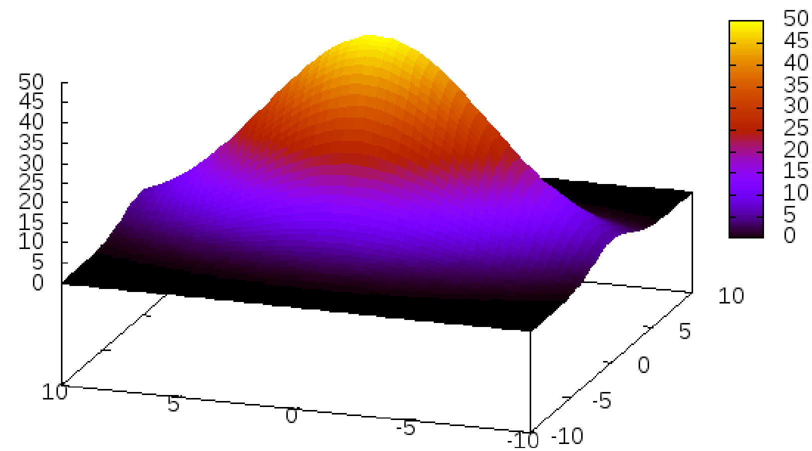
$\epsilon_2 = 0.2$



$\epsilon_2 = 0.4$



$\epsilon_2 = 0.6$



$$T^{00} = A \cdot \exp\left(\frac{-r^2\left(1+\overline{\epsilon_2}\cdot\cos\left(2\varphi\right)\right)}{2\cdot B^2}\right)$$

Definition of eccentricity:

$$\epsilon_2 = -\frac{\left\langle r^2 e^{i2\varphi}\right\rangle}{\left\langle r^2\right\rangle} = -\frac{\iint_{-\infty}^{\infty} r^2 e^{i2\varphi} T^{00}\, dxdy}{\iint_{-\infty}^{\infty} r^2 T^{00}\, dxdy}$$

Solved analitically:

$$\epsilon_2 = \overline{\epsilon_2}$$

Valid for any $\overline{\epsilon_2}$ independent of x and y

Rapidity is defined as follows:

$$Y = atanh\left(\frac{p_z}{E}\right)$$

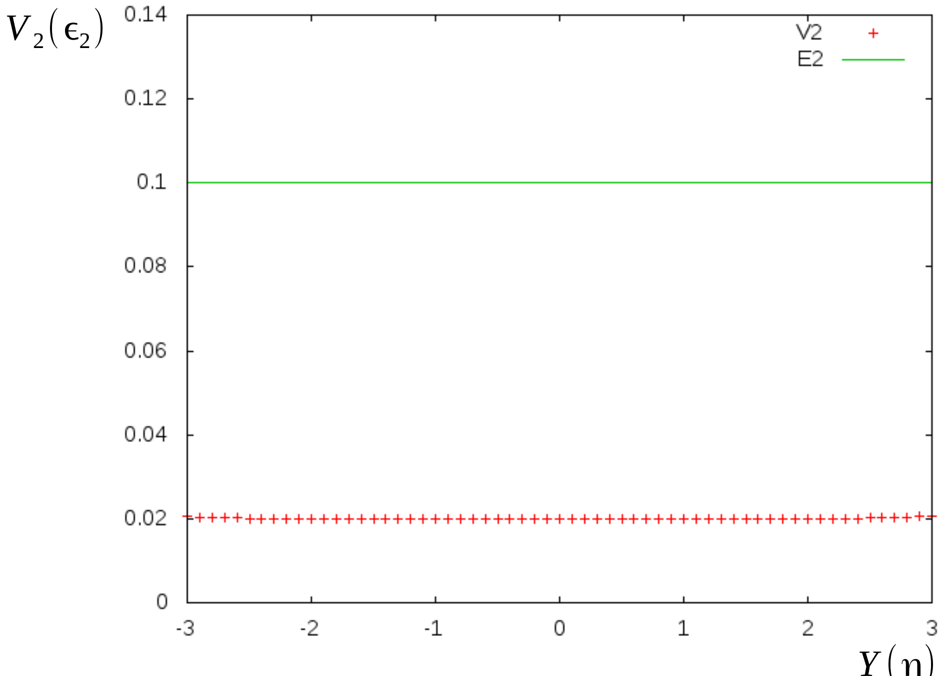
We can use spacetime rapidity:

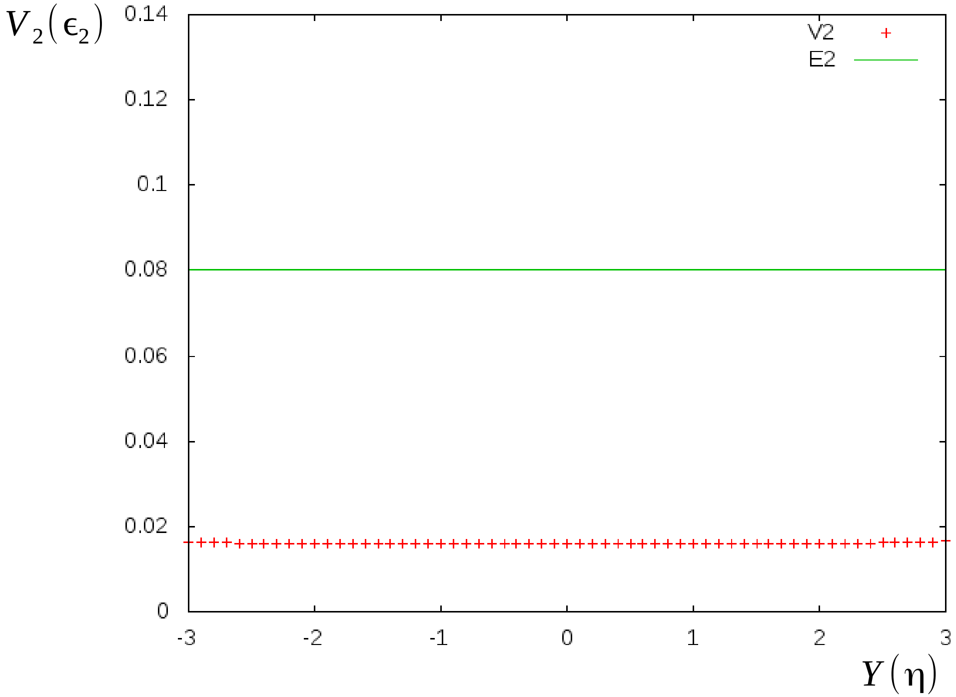
$$\eta = atanh\left(\frac{z}{t}\right)$$

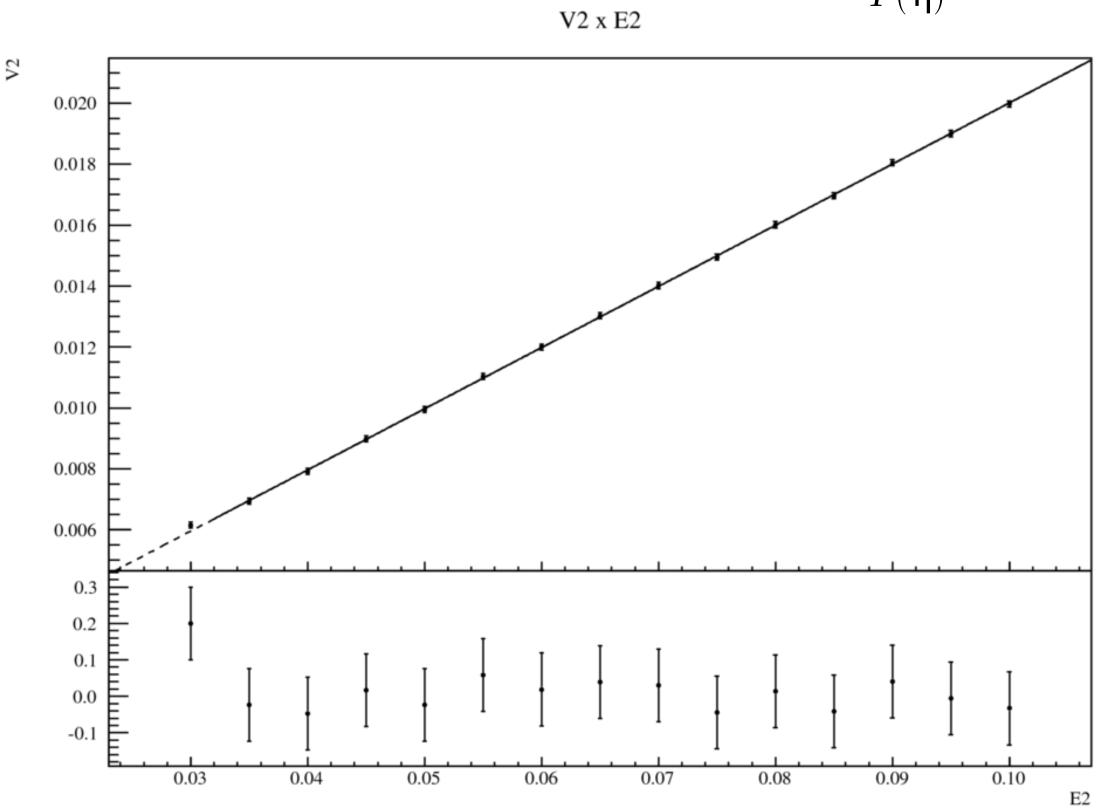
Boost invariant system

- Independent of Y
- Independent of η

$$\epsilon_2 = \epsilon_2(\eta)$$







Boost Invariant

$$\epsilon_2 = a$$

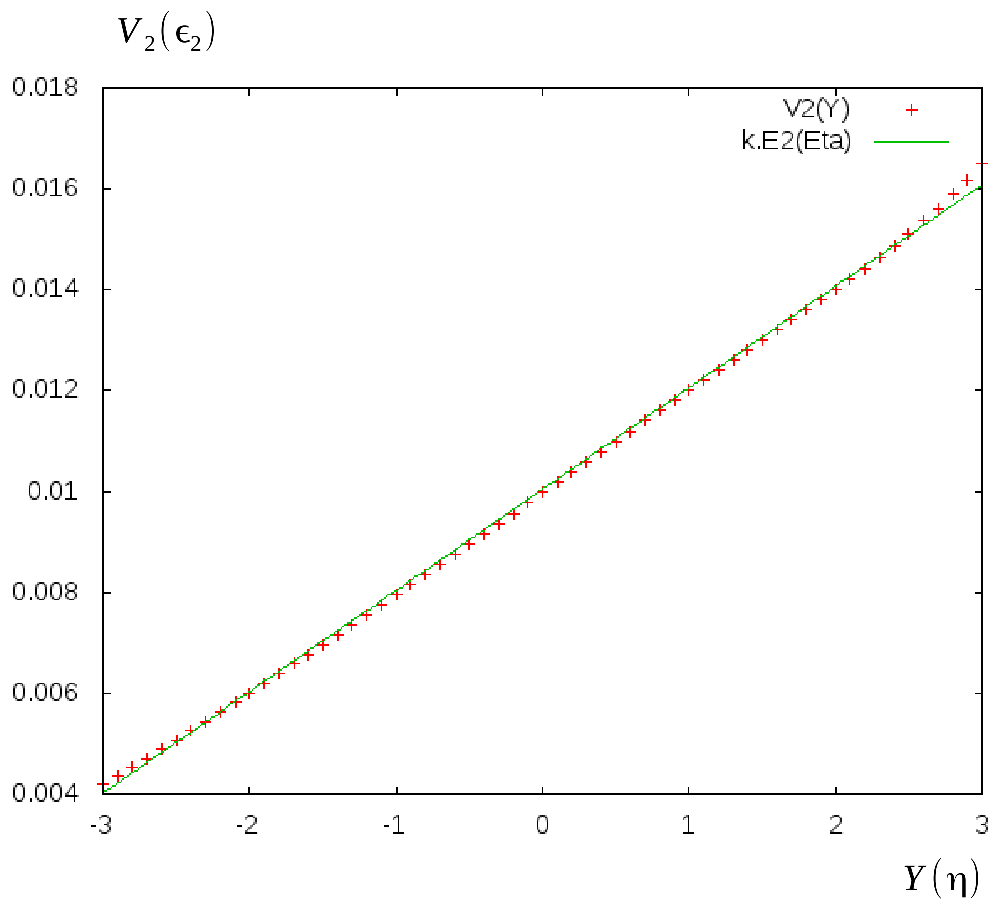
$$V_2 = k \cdot \epsilon_2$$

k = 0.201 (1)

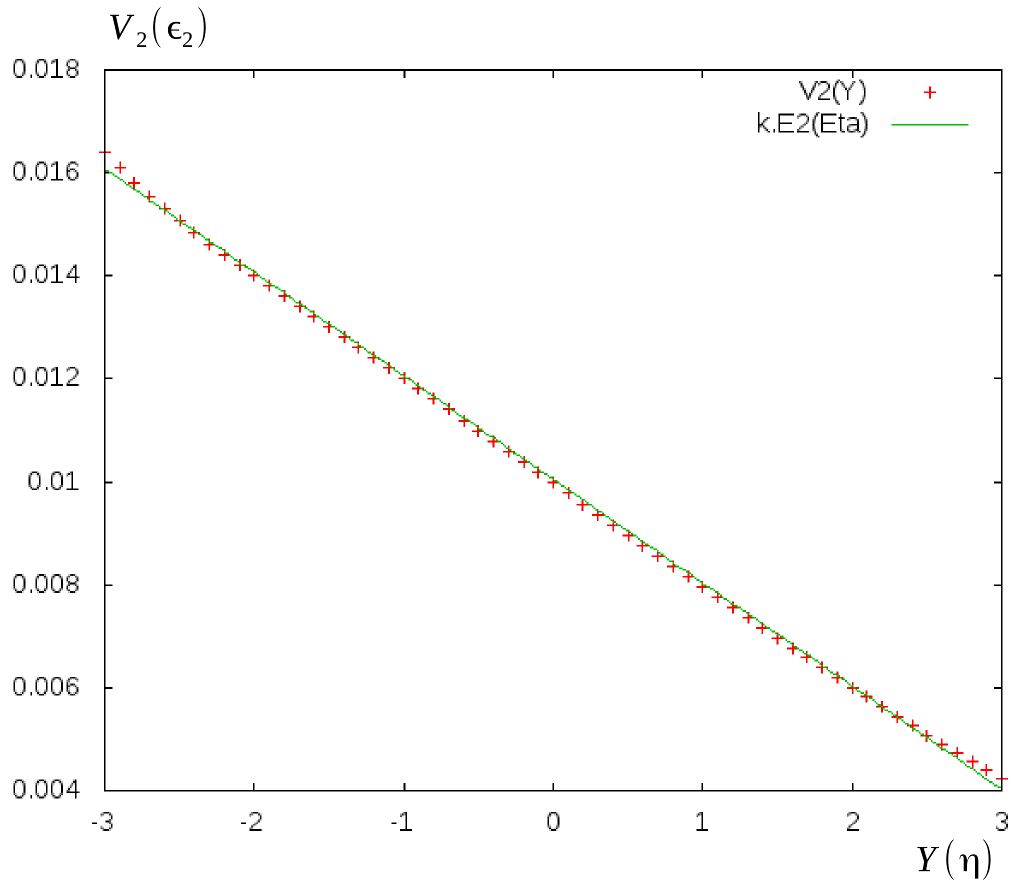
$$\epsilon_2(\eta) = a \cdot \eta + b$$

$$V_2(Y) = k \cdot \epsilon_2(\eta)$$
$$k = 0.201$$

a = 0.01 b = 0.05



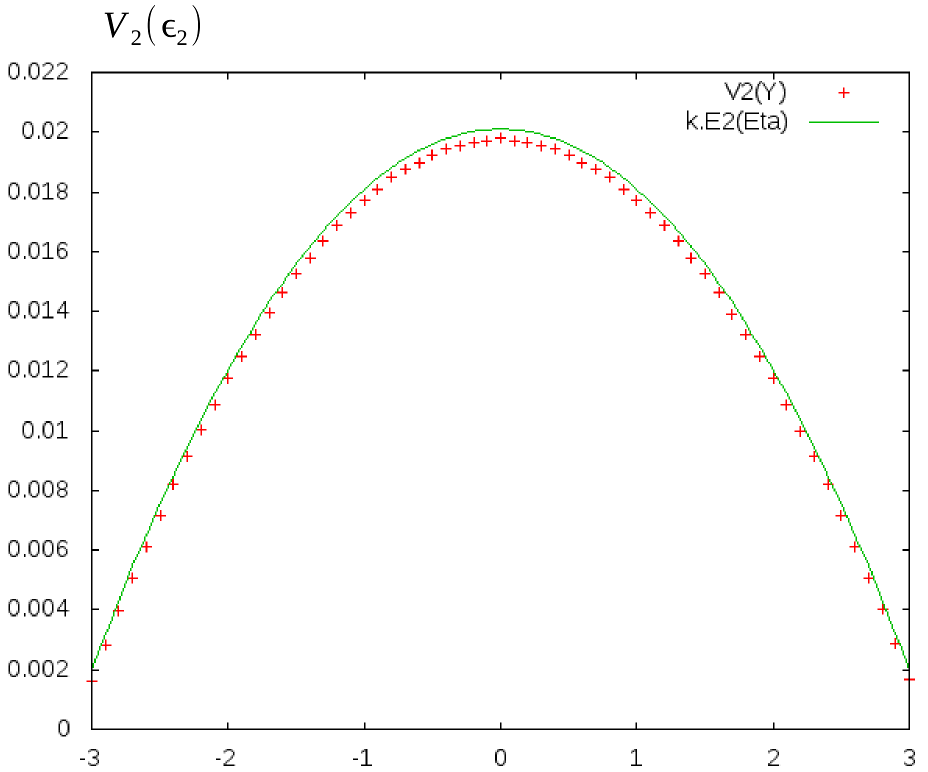
a = -0.01 b = 0.05



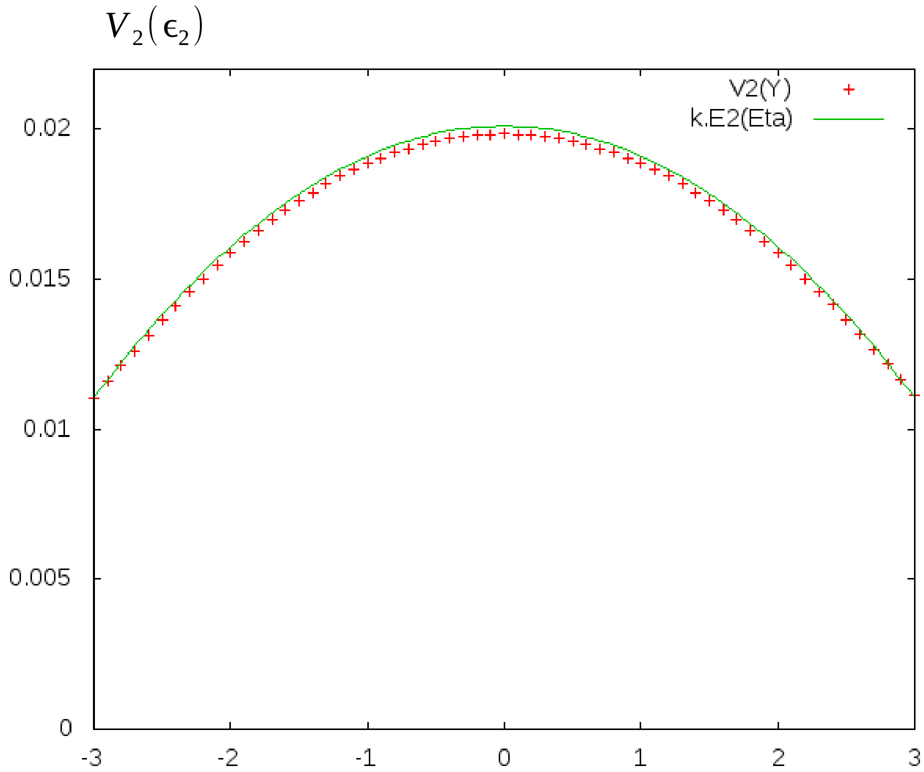
$$\epsilon_2(\eta) = -a \cdot \eta^2 + b$$

$$V_2(Y) = k \cdot \epsilon_2(\eta)$$
$$k = 0.201$$

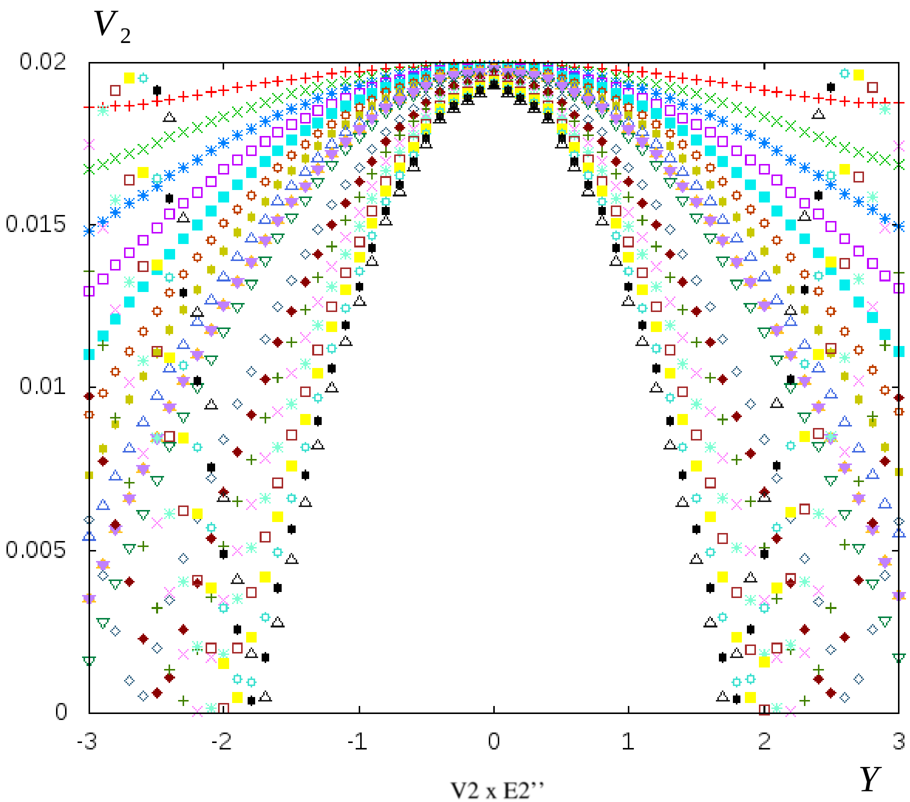
a = 0.01 b = 0.1

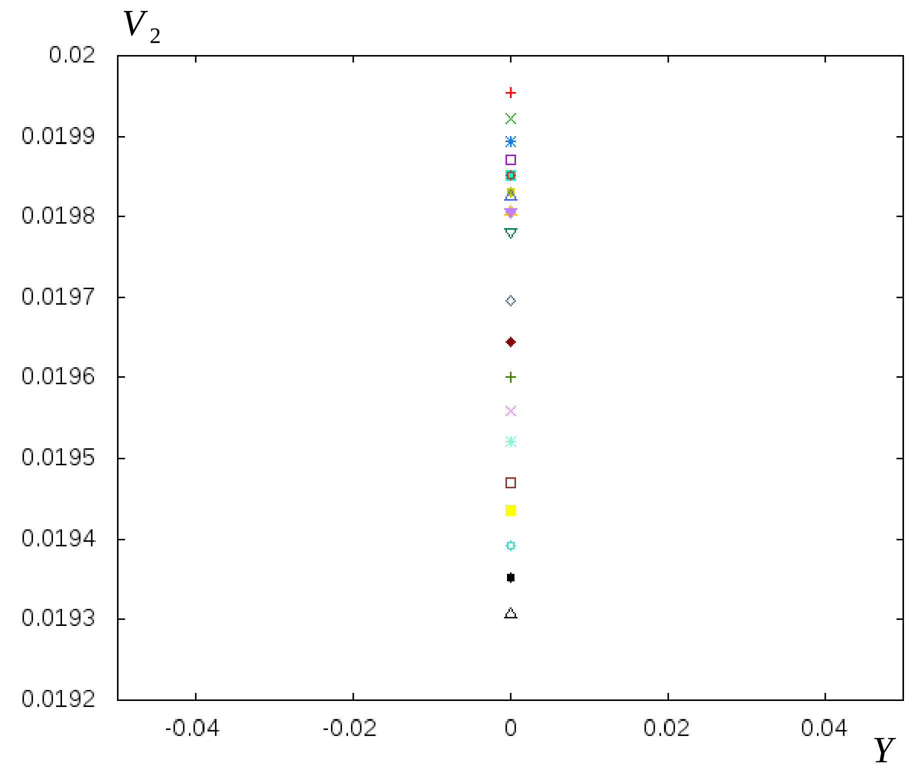


a = 0.005 b = 0.1

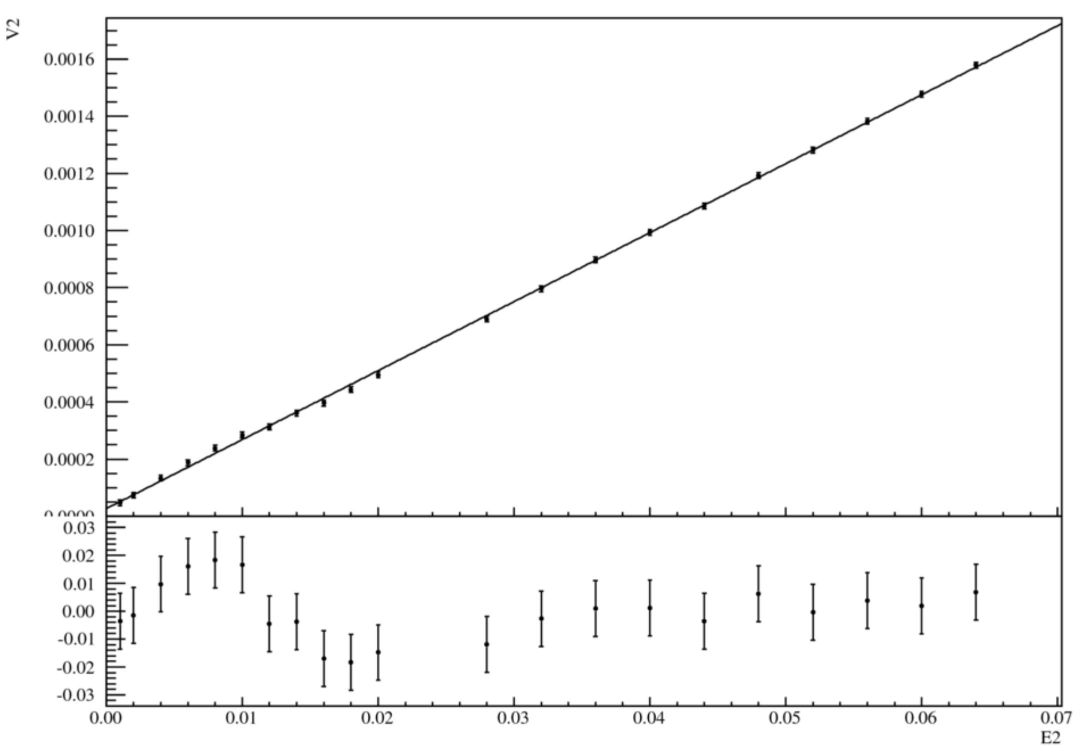


What can we do to improve our relation?





V2 x E2''



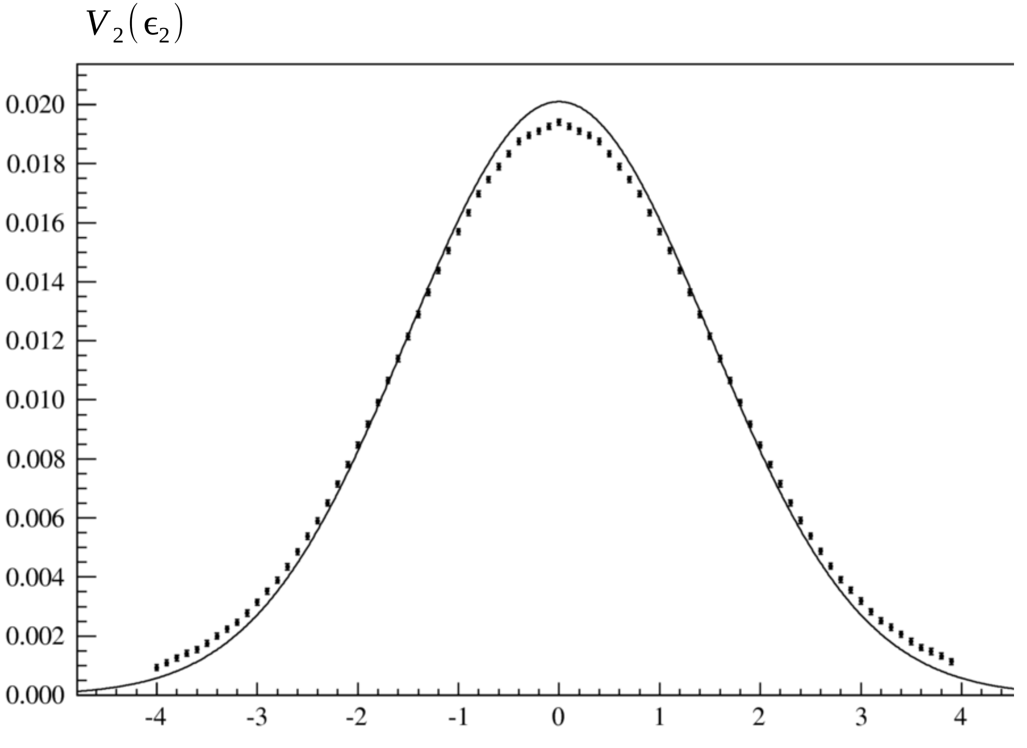
$$\epsilon_2(\eta) = -a \cdot \eta^2 + b$$

$$V_2(Y=0) - k \cdot \epsilon_2(\eta=0) = k'' \epsilon_2''(\eta=0)$$

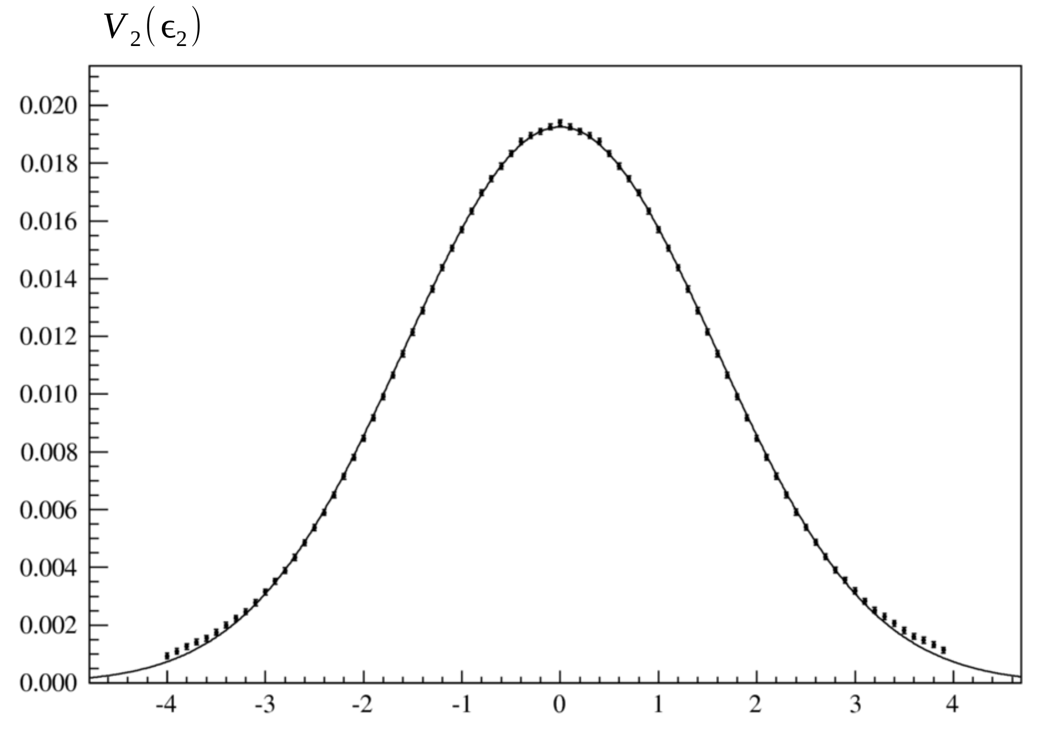
$$k'' = 0.0241(1)$$

$$\epsilon_2(\eta) = A \cdot e^{-\frac{\eta^2}{2 \cdot sig^2}}$$

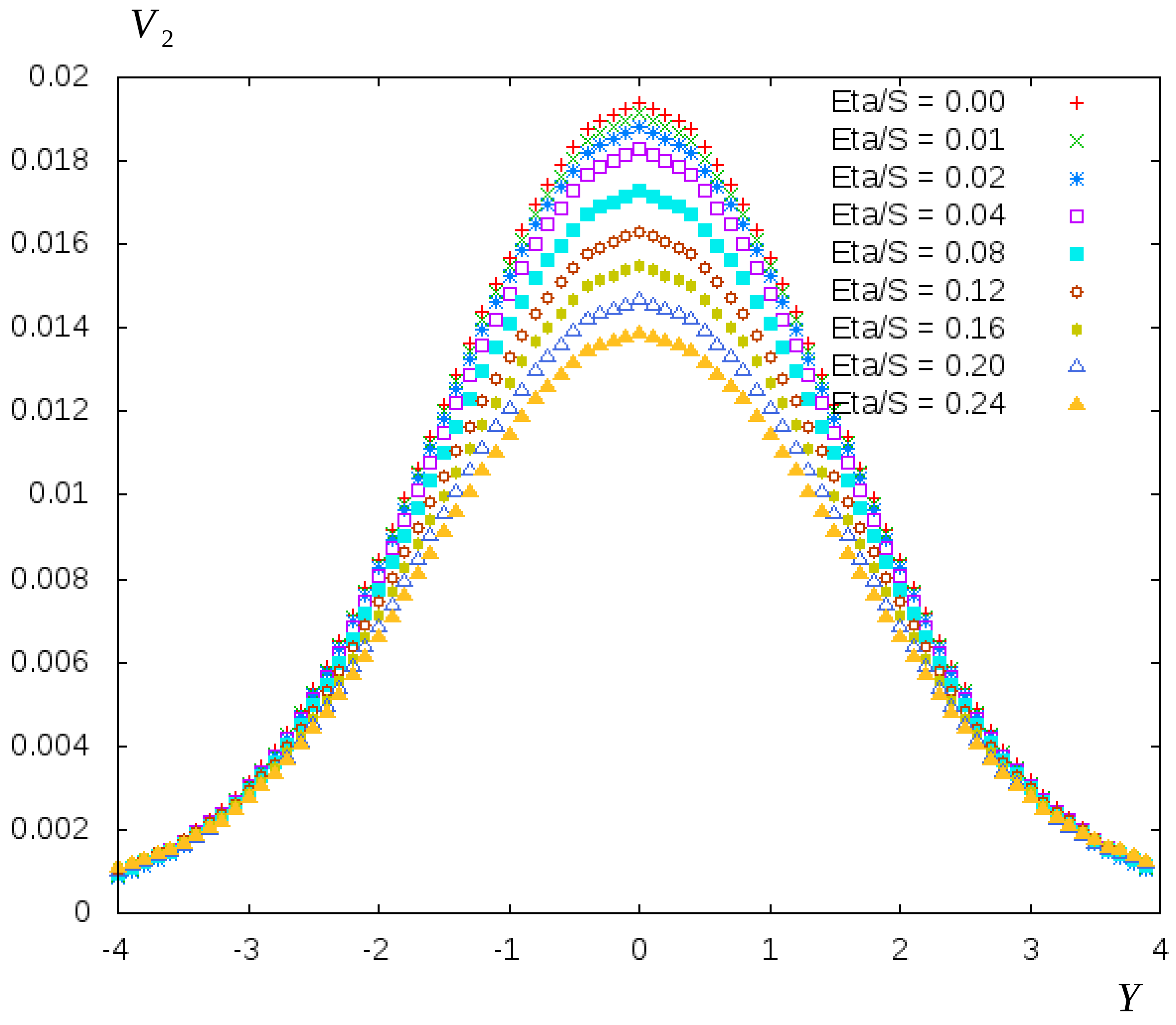
$V_2(Y) = k \cdot \epsilon_2(\eta)$

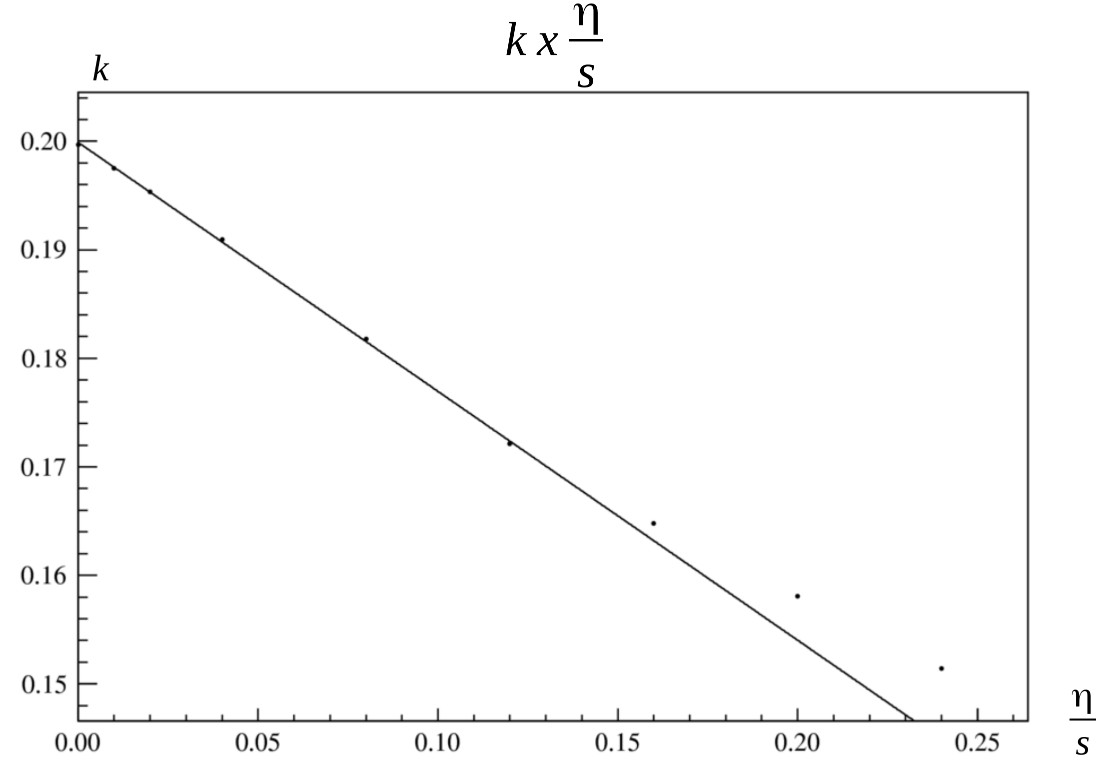


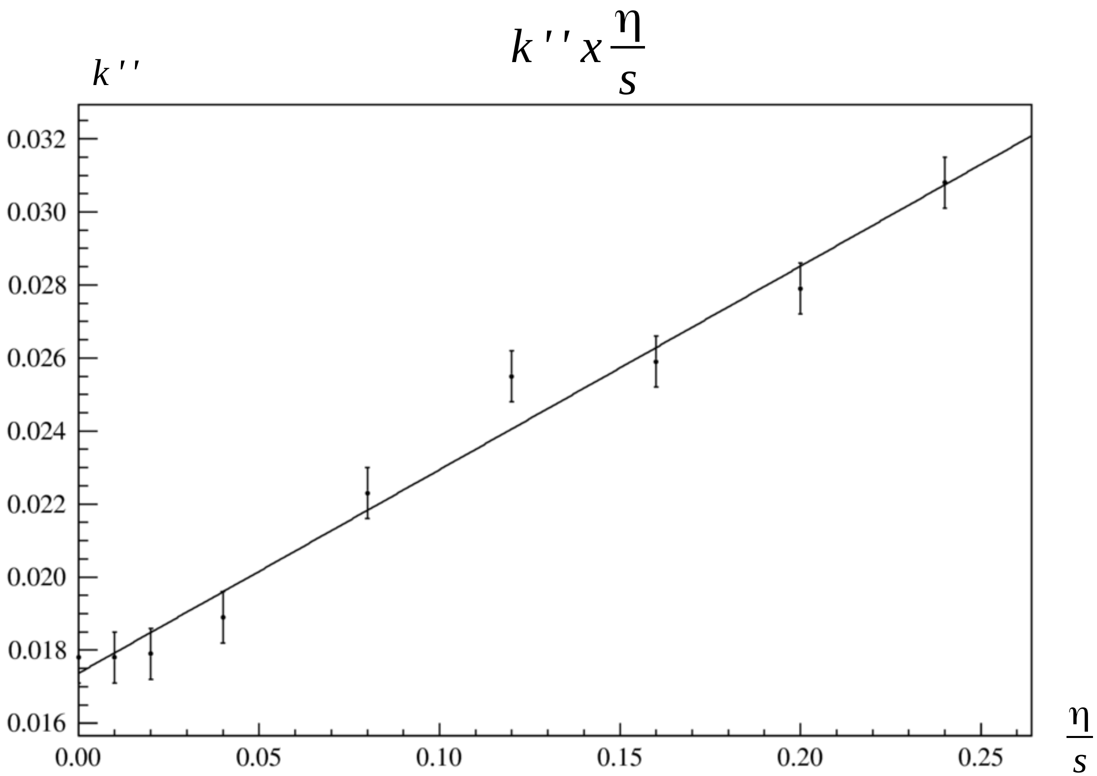
$V_2(Y) = k \cdot \epsilon_2(\eta) + k'' \cdot \epsilon_2''(\eta)$



Constants K dependence on shear viscosity







$$V_2(Y) = k \cdot \epsilon(\eta) + k'' \cdot \epsilon''(\eta)$$

Conclusions

- Linear coefficient is sufficient
- Addition of second derivative
- Gaussian plots to obtain the constants
- Improve results

Next Steps

- Study different flows
- Study more realistic initial conditions
- Energy-momentum tensor components