Mapping the principal component flow analysis from initial conditions

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November 9, 2018

FAPESP

Hydrodynamic response in heavy-ion collisions

- Anisotropic *flow* is an invaluable probe of the QGP.
- A controlled *mapping* of the effect of initial conditions is possible.
- To what extent is this mapping reliable, what kind of information is kept or lost?
- Ultimate visualization of two-particle correlations: **PCA**.
- What does the second PCA mode tell us?

F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).
R. S. Bhalerao, J. Y. Ollitrault, S. Pal and D. Teaney, PRL **114** (2015).

Anisotropic flow

We define $\mathcal{V}$ and V such that

$$\frac{dN}{p_T\, dp_T\, d\eta\, d\varphi} = \frac{1}{2\,\pi} \frac{dN}{p_T\, dp_T\, d\eta} \sum_{n=-\infty}^{\infty} V_n(p_T, \eta)\, e^{-in\varphi} \tag{1}$$

$$= \frac{1}{2\,\pi} \sum_{n=-\infty}^{\infty} \mathcal{V}_n(p_T, \eta)\, e^{-in\varphi}\,, \tag{2}$$

While $\langle \mathcal{V}_{n\neq 0} \rangle_{\text{isotropic}} = 0$, angular correlations contained in

$$\mathcal{V}_{n\,\Delta}^{ab} := \langle \mathcal{V}_n^{a*} \mathcal{V}_n^{b} \rangle = \langle \mathcal{V}_n(p_T^a)^* \mathcal{V}_n(p_T^b) \rangle\,. \tag{3}$$

Mapping from initial conditions

Assumptions

- Evolution given by $T^{00}(\vec{x}, \tau_0) := \rho(\vec{x}, \tau_0)$.
- Large scale structure dominates.

Cumulant-generating function:

$$W(\vec{k}) := \log\left(\int d^2x\, \rho(\vec{x})\, e^{i\,\vec{k}\cdot x}\right) := \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} W_{n,m}\, k^m\, e^{-i\,n\,\phi_{\vec{k}}}\,, \tag{4}$$

$$V_n = V_n[\{W_{n,m}\}]\,. \tag{5}$$

F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).
D. Teaney and L. Yan, PRC **83** (2011).

Mapping from initial conditions

Since under rotations, $W_{n,m} \to W_{n,m}\, e^{i\,n\,\delta}$, we estimate V_n^a as

$$V_n^{a\,\text{est.}} \approx \sum_{m=n}^{m_{max}} \kappa_{n,m}^a\, \epsilon_{n,m} + \sum_{l=1}^{m_{max}} \sum_{m=l}^{m_{max}} \sum_{m'=|m-l|}^{m_{max}} \kappa_{l,m,m'}^a\, \epsilon_{l,m}\, \epsilon_{n-l,m'} + \dots\,, \tag{6}$$

where we define the eccentricities

$$\epsilon_{n,m} := \frac{W_{n,m}}{(W_{0,2})^{m/2}} \tag{7}$$

and κ^a is found from minimizing

$$(\mathcal{E}^a)^2 := \langle |V_n^{a\,\text{est.}} - V_n^a|^2 \rangle\,. \tag{8}$$

Principal Component Analysis (PCA)

Full $\mathcal{V}^{ab}_{n\,\Delta} = \mathcal{V}_{n\,\Delta}(p^a_T, p^b_T)$ can be a large matrix. How to visualize it?

Spectral Theorem

Since $\mathcal{V}_{n\,\Delta} = (\mathcal{V}_{n\,\Delta})^\dagger$, decomposition in eigenvalues

$$\mathcal{V}^{ab}_{n\,\Delta} = \sum_i \lambda^{(i)}_n\, \psi^{(i)}_n(p^a_T)\, \psi^{(i)}_n(p^a_T)^*\,, \tag{9}$$

which can be truncated for $\lambda^{(i)}_n \gg \lambda^{(i+1)}_n$.

Useful definition:

$$v^{(i)}_n(p^a_T) := \sqrt{\lambda^{(i)}_n}\, \psi^{(i)}_n(p^a_T)/\langle N(p^a_T)\rangle \tag{10}$$

R. S. Bhalerao, J. Y. Ollitrault, S. Pal and D. Teaney, PRL **114** (2015).

Principal Component Analysis (PCA)

Since $\mathcal{V}_{n\,\Delta}$ is a covariance matrix, $\lambda_n^{(i)}$ are related to the amplitude and profile of the leading fluctuations! Each mode fluctuates *independently*.

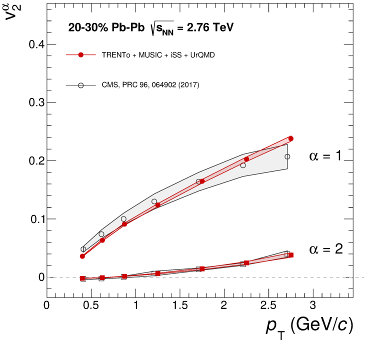
Examples

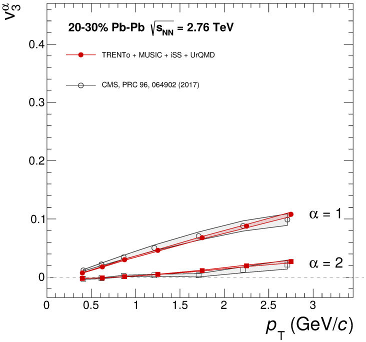
- For momentum-independent fluctuations $\psi^{(i)}(p_T^a) = \text{const.}$
- For uncorrelated momentum bins $\psi^{(i)}(p_T^a) = \delta^{\text{Kronecker}}_{p_T^a\, p_T^{(i)}}$.

$\mathcal{V}_{n\,\Delta}$ contains the *full information* on two particle angular correlations! The leading $\lambda_n^{(i)}$ contain *most* of it!

Hydrodynamic simulations

- TRENTo + MUSIC + iSS Sampler + UrQMD + Hadrex.
- Here $Pb + Pb$ @ $\sqrt{s_{NN}} = 2.76$ TeV. Next, $\sqrt{s_{NN}} = 5.02$ TeV.





T. Nunes - recent presentation.

Mapping differential flow

- We have mapped V_n^a from $\epsilon_{n,m}$, with $n = 1 - 5$, $m = 2 - 5$ and different numbers of terms.

- Success measured by the Pearson correlation coefficient:

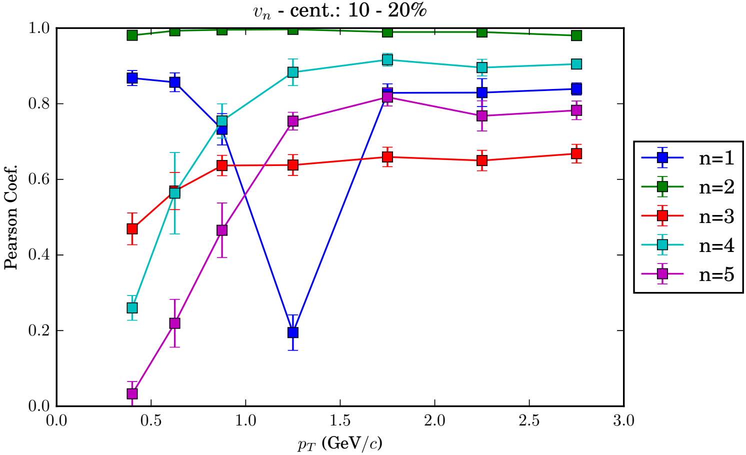
$$\frac{\langle V_n^* \, V_n^{a\,\mathrm{est.}} \rangle}{\sqrt{\langle |V_n|^2 \rangle \, \langle |V_n^{a\,\mathrm{est.}}|^2 \rangle}} \leq 1 \,.$$

- $V_n^{\mathrm{est.}}$ from initial conditions, V_n from after freezeout.

- Nonlinear terms were included.

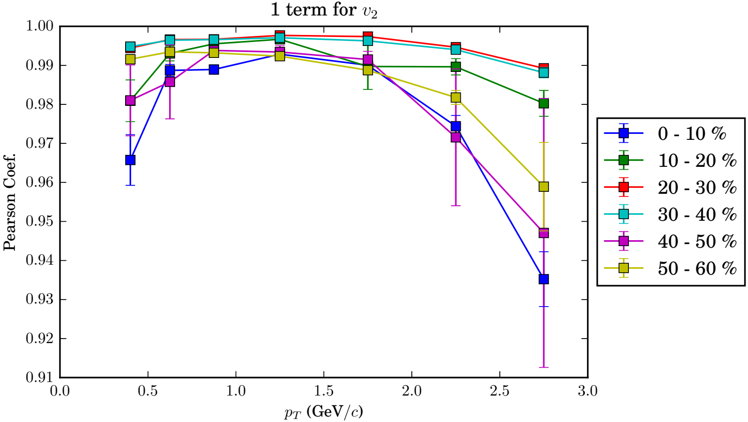
- From now on, we focus on v_2.

Mapping differential flow - 1 term



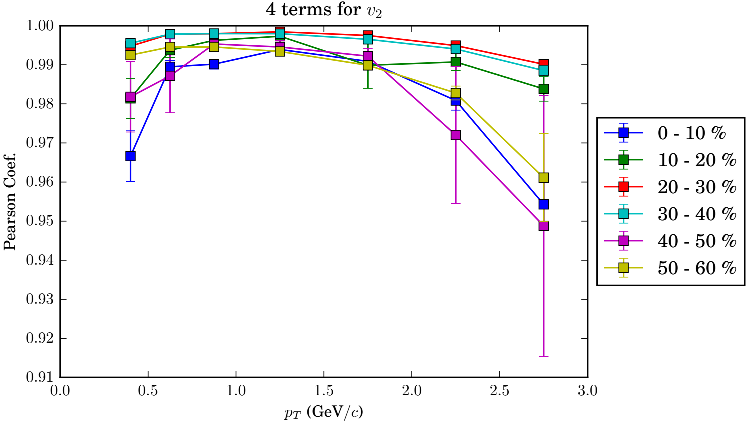
Momentum and centrality dependent. Eccentricities $\epsilon_{n,n}$ and $\epsilon_{1,3}$.

Mapping differential flow - 1 term



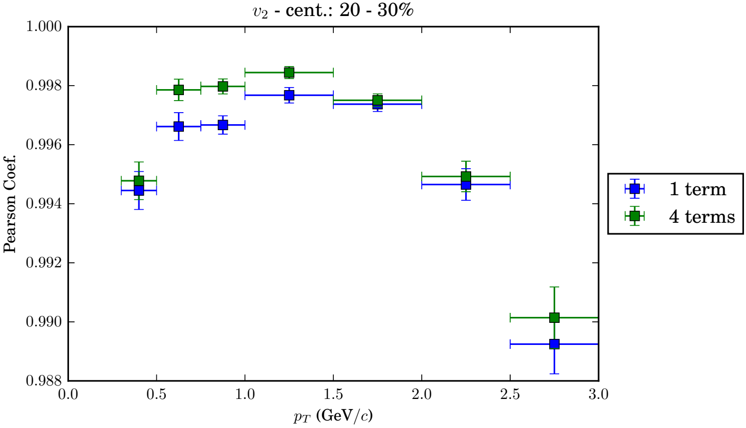
Momentum and centrality dependent. Eccentricities $\epsilon_{n,n}$ and $\epsilon_{1,3}$.

Mapping differential flow - 4 terms



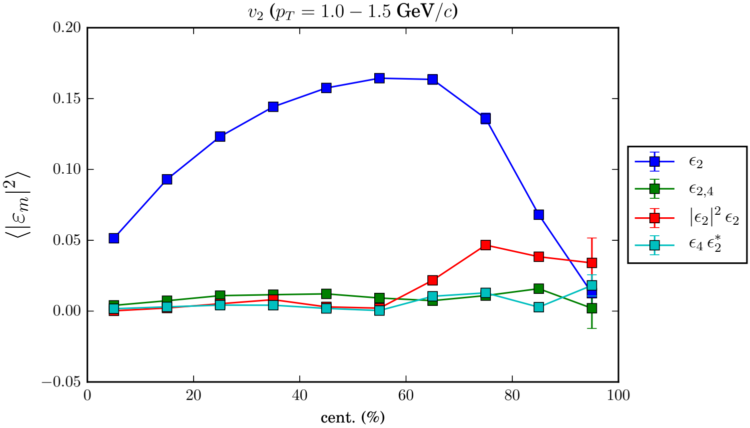
Extra terms do not enhance the performance much.

Improvement - v_2



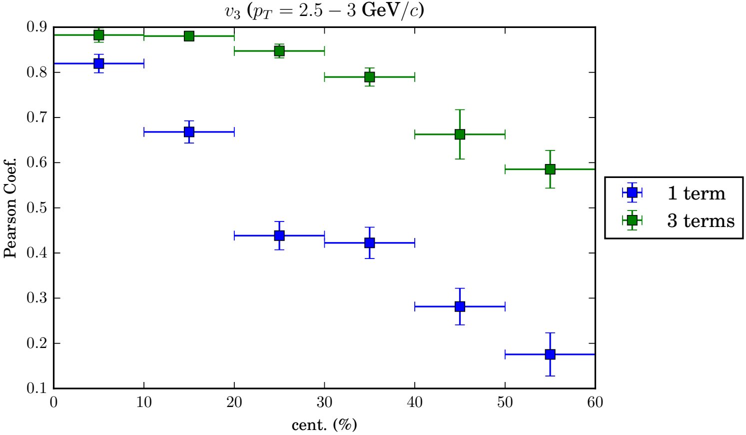
Extra terms do not enhance the performance much.

Relevance of each term - v_2 (variance)



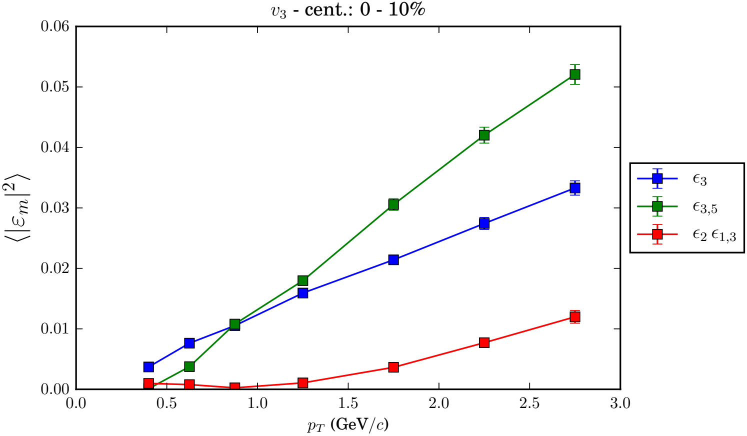
Strongly dominating term.

Improvement - v_3



Sometimes, quite the opposite.

Relevance of each term - v_3 (variance)

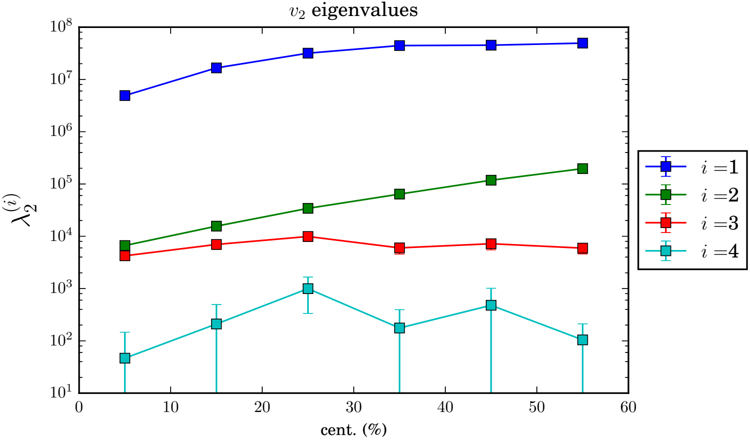


In fact, terms might compete.

Principal component analysis

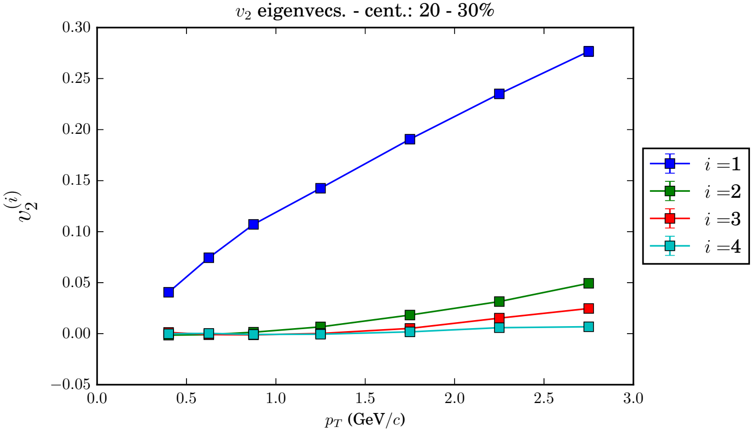
- We have also obtained the v_n PCA for $n = 0 - 5$ from the simulations.
- Prediction already presented by T. Nunes.
- Again, we focus on v_2.

Principal component analysis - eigenvalues



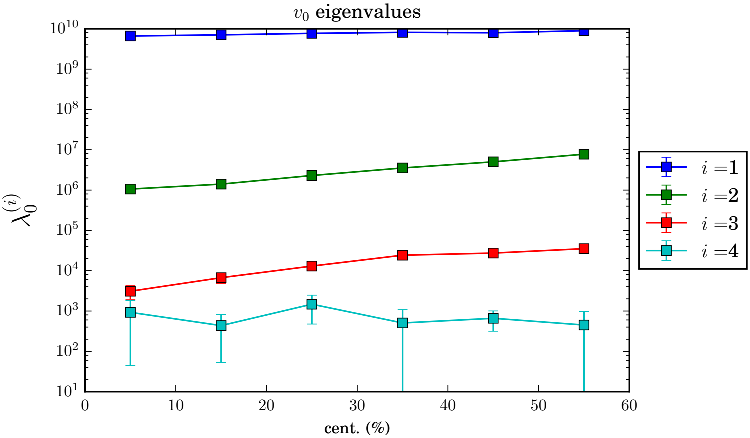
Strong hierarchy! (Log scale)

Principal component analysis - modes



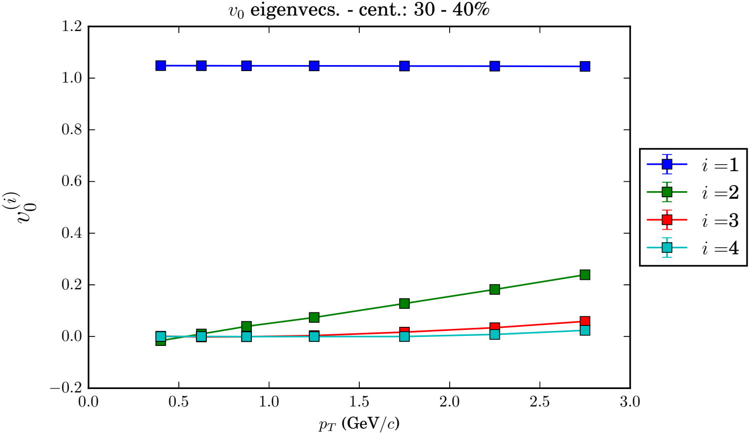
Correlation profile for each mode.

Multiplicity PCA eigenvalues



Multiplicity per bin also fluctuates in several ways.

Multiplicity PCA eigenvectors



$\langle N \rangle^2$ dominates $\mathcal{V}_{0\,\Delta}$! Remember normalization $\sim \sqrt{\lambda}\,\langle N^a \rangle^{-1}$.

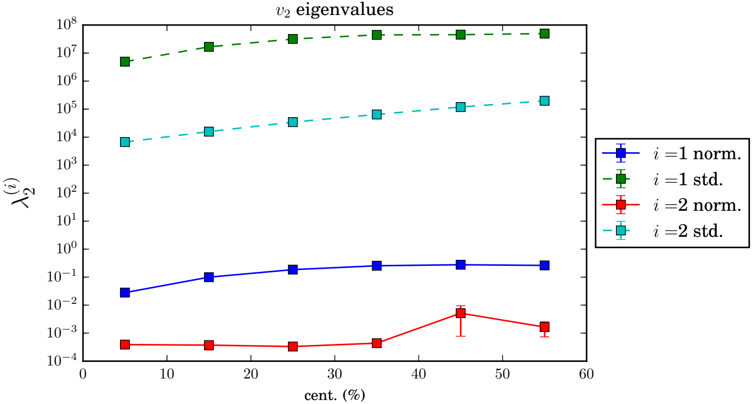
Normalized PCA scheme

- Multiplicity fluctuations have nontrivial structure.
- Scalar fluctuations not covered by the mapping, but $\mathcal{V}^a = V^a\, N^a$.
- 2-particle angular correlations in

$$V_{n\,\Delta} := \left\langle \exp\left[i\,n\,\left(\varphi^{(a)} - \varphi^{(b)}\right)\right] \right\rangle_{\text{pairs}} = \left\langle V_n^{a*}\, V_n^b \right\rangle . \qquad (11)$$

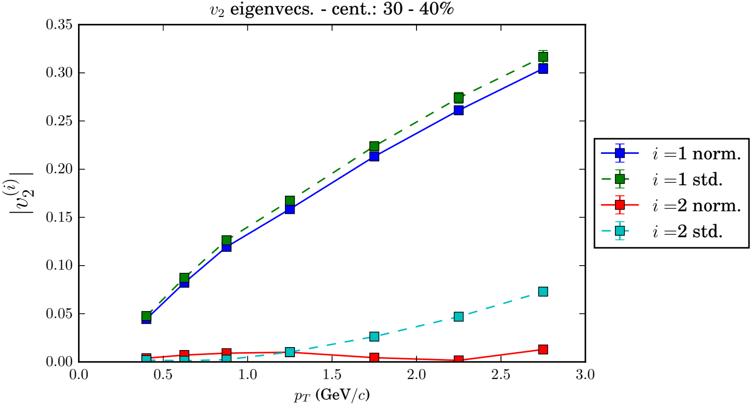
- Normalized PCA scheme! $\mathcal{V}^a \Rightarrow V^a$

Normalized PCA eigenvalues



Still a strong hierarchy.

Normalized PCA modes

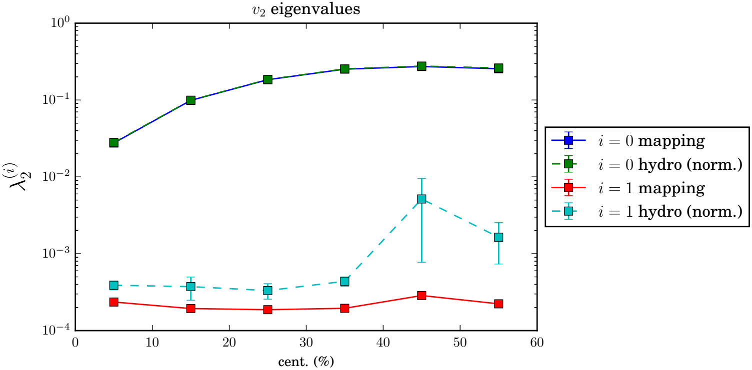


More effects at high p_T and peripheral collisions – fewer particles?

Mapping the PCA

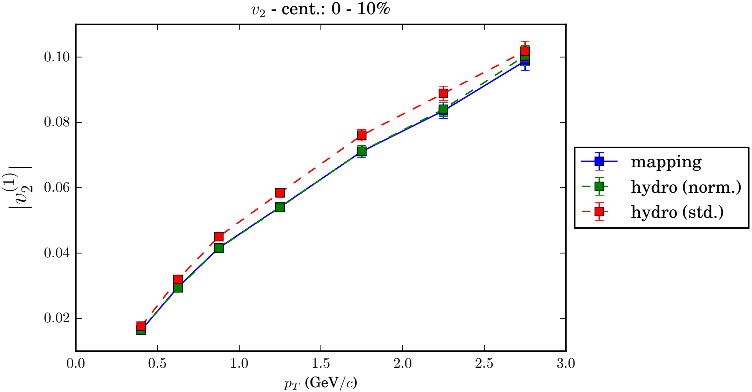
- We now apply the mapping to reproduce the first two PCA modes.
- The 2^{nd} PCA mode is a quite sensitive observable (fine detail).
- Comparison is more sensible within the normalized scheme.
- "Ultimate test" for 2-particle correlations.

Mapping the PCA eigenvalues



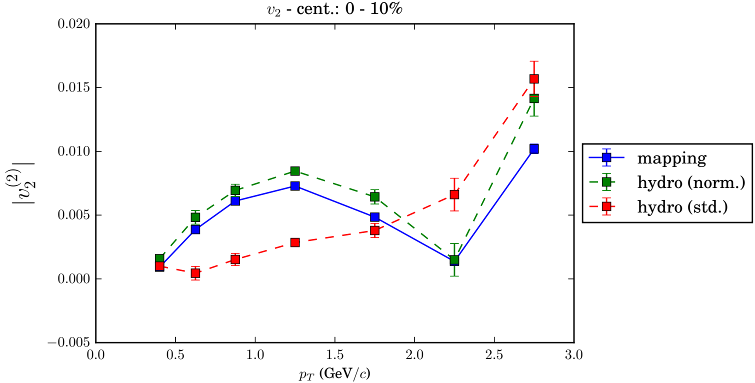
Not impressive, but eigenvalues depend on the overall structure.

Mapping the 1st PCA mode



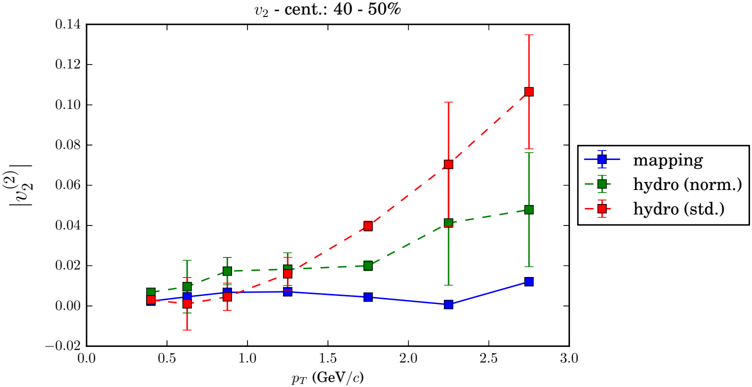
The first component is usually quite robust.

Mapping the 2nd PCA mode - “the good”



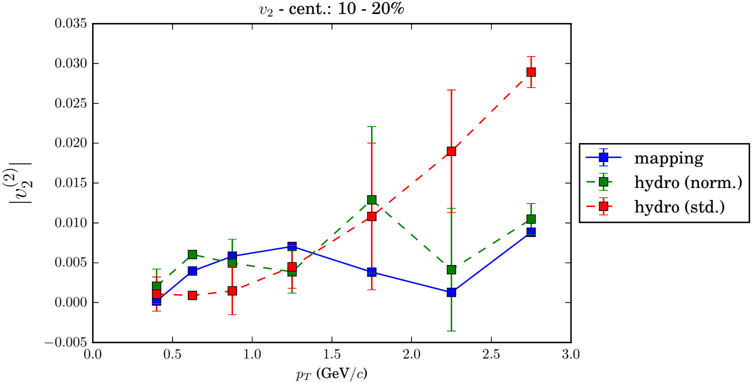
Sometimes, the mapping nearly reproduces the second mode.

Mapping the 2nd PCA mode - "the bad"



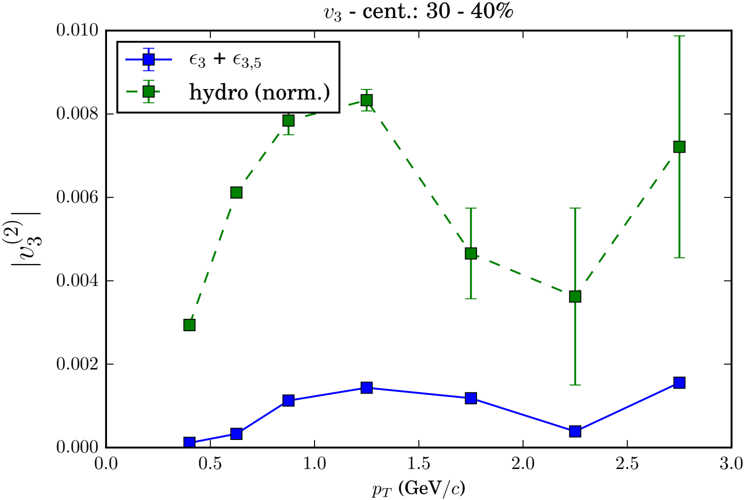
Sometimes a little bit worse.

Mapping the 2nd PCA mode - "the ugly"



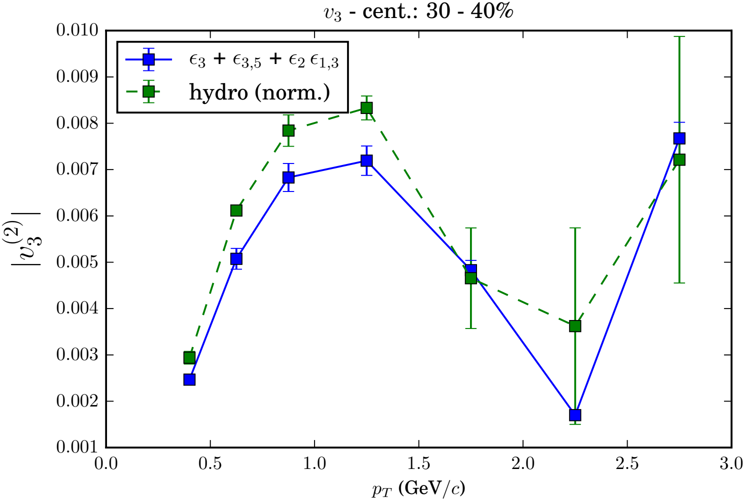
Sometimes it looks better than the full results. (?)

Terms *vs* features: v_3 - 2 terms



Looking elsewhere...

Terms *vs* features: v_3 - 3 terms



The effects of extra terms can be quite impressive.

Summary

- The mapping of hydrodynamic response was reviewed.
- The principal component analysis (PCA) as well.
- A normalized PCA scheme was developed.
- The PCA was employed as a new test of the mapping.

Final remarks

- The PCA is the ultimate test based on pairwise correlations.
- The mapping seems to $\sim$ work for v_2.
- Stricter kinematic cuts might enhance performance.
- Large number of terms might be required.
- More statistics required.
- So far, promising results.

Acknowledgements

Thanks to FAPESP for financial support.

Cumulant examples

$$W_{0,2} = \frac{i^2}{2!}\frac{1}{2}\,\left[\{r^2\} - \{r\,e^{-i\,\phi}\}\,\{r\,e^{-i\,\phi}\}\right]\,, \tag{12}$$

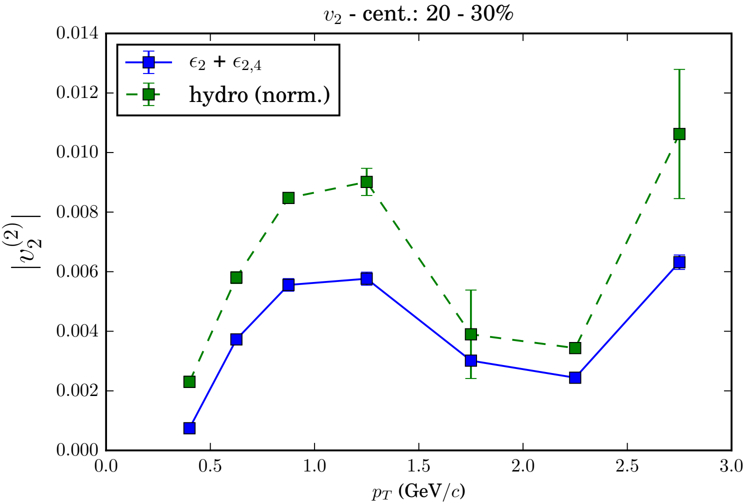
$$W_{2,2} = \frac{i^2}{2!}\frac{1}{4}\,\left[\{r^2\,e^{2\,i\,\phi}\} - \{r\,e^{i\,\phi}\}^2\right]\,, \tag{13}$$

$$W_{1,3} = \frac{i^3}{3!}\frac{3}{8}\,\left[\{r^3\,e^{i\,\phi}\} - \{r^2\,e^{2\,i\,\phi}\}\,\{r\,e^{-i\,\phi}\}\right]\,, \tag{14}$$

where

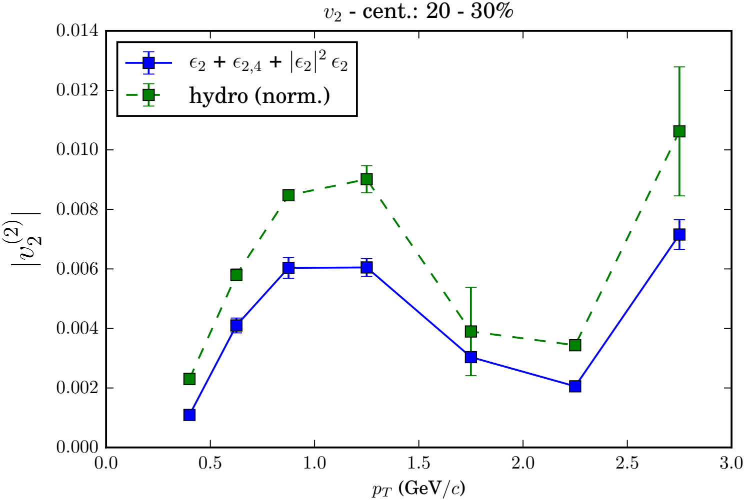
$$\{\ldots\} := \frac{\int d^2x\,\rho(\vec{x})\,(\ldots)}{\int d^2x\,\rho(\vec{x})} \tag{15}$$

v_2 - 2 terms



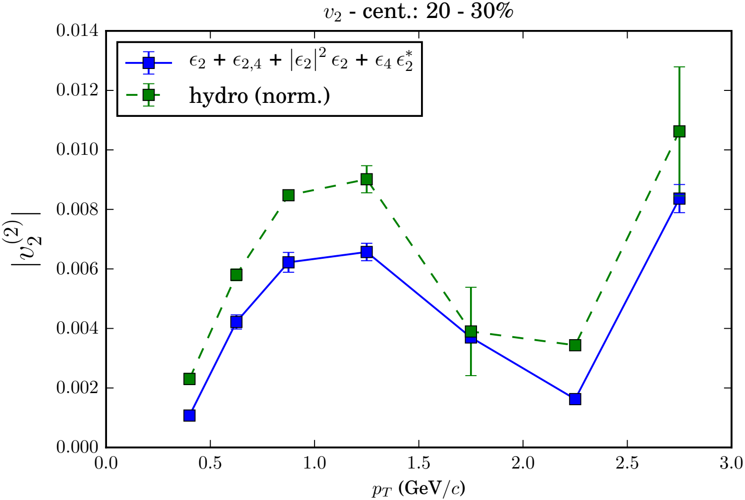
Beware: careful choice!

v_2 - 3 terms



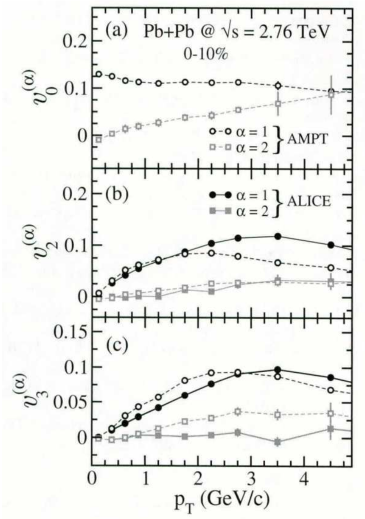
Beware: careful choice!

v_2 - 4 terms



Beware: careful choice!

Original PCA from AMPT



Factorization breaking

Notice that, if $\mathcal{V}_{n\,\Delta}$ has only one non-vanishing eigenvalue,

$$\mathcal{V}_{n\,\Delta}^{ab} = \sqrt{\lambda_n}\,\psi_n(p_T^a)\,\sqrt{\lambda_n}\,\psi_n(p_T^b)^*\,, \tag{16}$$

$$\left\langle \frac{dN}{d\vec{p}_1}\,\frac{dN}{d\vec{p}_2} \right\rangle = \left\langle \frac{dN}{d\vec{p}_1} \right\rangle \left\langle \frac{dN}{d\vec{p}_2} \right\rangle\,, \tag{17}$$

and the subleading modes are connected to factorization breaking.

Anisotropic flow′

We use the flow vector

$$Q_n^a = \sum_{i=1}^{N^a} e^{i\,n\,\varphi_i^{(a)}} \,. \tag{18}$$

While $\langle Q_{n\neq 0}\rangle_{\text{isotropic}} = 0$, angular correlations contained in

$$Q_{n\,\Delta}^{ab} = \langle Q_n^{a*} Q_n^b \rangle - \delta^{ab} \, \langle Q_0^a \rangle \neq 0 \,. \tag{19}$$

If we write

$$\frac{dN}{p_T \, dp_T \, d\eta \, d\varphi} = \frac{1}{2\,\pi} \frac{dN}{p_T \, dp_T \, d\eta} \sum_{n=-\infty}^{\infty} V_n(p_T, \eta) \, e^{-in\varphi} \,, \tag{20}$$

then $V_n^a = Q_n^a / N^a$ and $v_n^a = |V_n^a|$.