

Homework Set 4 — Scattering 1

Due May 12, 2021

1. Given a cross section of

$$\frac{d\sigma}{d\Omega} = 10^{-34} \sin^2 \theta \text{ cm}^2, \quad (1)$$

compute the number of particles per unit time scattering into a detector with circular cross-section area 10 m^2 located a distance 100 m from the scattering center off-axis at an angle of 30° from the beam axis that is exposed to a beam of 2×10^{32} particles per unit time per cm^2

2. The Lippmann-Schwinger formalism can also be applied to a *one*-dimensional transmission-reflection problem with a finite-range potential, $V(x) \neq 0$ for $0 < |x| < a$ only.

(a) Suppose we have an incident wave coming from the left: $\langle x|\phi\rangle = e^{ikx}/\sqrt{2\pi}$. How must we handle the singular $1/(E - H_0)$ operator if we are to have a transmitted wave only for $x > a$ and a reflected wave and the original wave for $x < -a$? Is the $E \rightarrow E + i\epsilon$ prescription still correct? Obtain an expression for the appropriate Green's function and write an integral equation for $\langle x|\psi^{(+)}\rangle$.

(b) Consider the special case of an attractive δ -function potential

$$V = - \left(\frac{\gamma \hbar^2}{2m} \right) \delta(x), \quad (\gamma > 0). \quad (2)$$

Solve the integral equation to obtain the transmission and reflection amplitudes.

(c) The one-dimensional δ -function potential with $\gamma > 0$ admits one (and only one) bound state for any value of γ . Show that the transmission and reflection amplitudes you computed have bound-state poles at the expected positions when k is regarded as a complex variable.

3. Consider the 1D step-up potential

$$V(r) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases} \quad (3)$$

with $V_0 > 0$. A particle with kinetic energy $E > V_0$ is incident from the left.

- (a) Find the intensity of the reflected (R) and transmitted (T) waves.
- (b) Find the currents (i.e., the intensities multiplied by the velocities) J_R and J_T of the reflected and transmitted waves and their sum $J_R + J_T$.

4. (a) Prove

$$\frac{\hbar^2}{2m} \langle \mathbf{x} | \frac{1}{E - H_0 + i\epsilon} | \mathbf{x}' \rangle = -ik \sum_l \sum_m Y_l^m(\hat{r}) Y_l^{m*}(\hat{r}') j_l(kr_<) h_l^{(1)}(kr_>), \quad (4)$$

where $r_<(r_>)$ stands for the smaller (larger) of r and r' .

- (b) For spherically symmetric potentials, the Lippmann-Schwinger equation can be written for *spherical* waves:

$$|Elm(+)\rangle = |Elm\rangle + \frac{1}{E - H_0 + i\epsilon} V |Elm(+)\rangle. \quad (5)$$

Using (4), show that this equation, written in the \mathbf{x} -representation, leads to an equation for the radial function, $A_l(k; r)$, as follows:

$$A_l(k; r) = j_l(kr) - \frac{2mik}{\hbar^2} \int_0^\infty j_j(kr_<) h_l^{(1)}(kr_>) V(r') A_l(k; r') r'^2 dr'. \quad (6)$$

By taking r very large, also obtain

$$f_l(k) = e^{i\delta_l} \frac{\sin \delta_l}{k} \quad (7)$$

$$= - \left(\frac{2m}{\hbar^2} \right) \int_0^\infty j_l(kr) A_l(k; r) V(r) r^2 dr \quad (8)$$