

# Homework Set 3 — Symmetries 2

Due April 18, 2021

- Let  $\psi(\mathbf{x}, t)$  be the wave function of a spinless particle corresponding to a plane wave in three dimensions. Show that  $\psi^*(\mathbf{x}, -t)$  is the wave function for the plane wave with the momentum direction reversed.
  - Let  $\chi(\hat{\mathbf{n}})$  be the two-component eigenspinor of  $\sigma \cdot \hat{\mathbf{n}}$  with eigenvalue  $+1$  (see Sakurai Ch. 3.2). Using the explicit form of  $\chi(\hat{\mathbf{n}})$  (in terms of the polar and azimuthal angles  $\beta$  and  $\alpha$  corresponding to direction  $\hat{\mathbf{n}}$ ) verify that  $-\sigma_2 \chi^*(\hat{\mathbf{n}})$  is the two-component eigenspinor with the spin direction reversed.
- What is the time-reversed state corresponding to  $\mathcal{D}(R)|j, m\rangle$ ?
  - Using the properties of time reversal and rotations, prove the matrix elements of the rotation operator satisfy

$$\mathcal{D}_{m'm}^{(j)}(R) \equiv \langle j, m' | \mathcal{D}(R) | j, m \rangle \quad (1)$$

$$= i^{-2(m'+m)} \mathcal{D}_{-m', -m}^{(j)}(R). \quad (2)$$

- Suppose a spinless particle is bound to a fixed center by a 3D potential  $V(\mathbf{x})$  that is so asymmetric, no energy level is degenerate. Use time-reversal invariance to prove

$$\langle \mathbf{L} \rangle = 0 \quad (3)$$

for any energy eigenstate. (This is known as **quenching** of orbital angular momentum.) If the wave function of such a nondegenerate eigenstate is expanded as

$$\sum_l \sum_m F_{lm}(r) Y_l^m(\theta, \phi), \quad (4)$$

what kind of phase restrictions do we obtain on  $F_{lm}(r)$ ?

4. The Hamiltonian for a spin 1 system is given by

$$H = AS_z^2 + B(S_x^2 - S_y^2). \quad (5)$$

Solve this problem *exactly* to find the normalized energy eigenstates and eigenvalues. (A spin-dependent Hamiltonian of this kind actually appears in crystal physics). Is this Hamiltonian invariant under time reversal? How do the normalized eigenstates you obtained transform under time reversal?