Homework Set 5 — Scattering 2

Due May 25, 2020

1. For the *non-local separable* potential

$$\langle \mathbf{r}'|V|\mathbf{r}\rangle = \lambda v(\mathbf{r})v(\mathbf{r}'), \tag{1}$$

with v vanishing at large radius, determine the integral equation of $\Psi^+(\mathbf{r})$ and solve as much as possible. Discuss the Born series for this potential and obtain the scattering amplitude in the first Born approximation.

2. Consider the 3D repulsive potential

$$V(r) = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases} \tag{2}$$

with $V_0 > 0$. A particle with energy $E = \hbar^2 k^2/2m < V_0$ is incident upon the potential.

   (a) Derive the phase shift for the s-wave.

   (b) How does the phase shift behave for $V_0 \to \infty$?

   (c) Derive the total cross section in the limit of very low energy

3. Consider a particle of mass m which scatters in 3D from a potential which is a shell at radius a:

$$V(r) = -C\delta(r-a). \tag{3}$$

Derive the s-wave expression for the scattering cross section in the limit of very low energy.

4. Prove for a central potential

$$\sigma_{\text{tot}} \simeq \frac{m^2}{\pi\hbar^4}\int d^3x \int d^3x' V(r)V(r')\frac{\sin^2 k|\mathbf{x}-\mathbf{x}'|}{k^2|\mathbf{x}-\mathbf{x}'|^2} \tag{4}$$

in each of the following ways.

(a) By integrating the differential cross section computed using the first-order Born approximation.

(b) By applying the optical theorem to the forward-scattering amplitude in the *second*-order Born approximation. [Note that $f(0)$ is real if the first-order Born approximation is used.]

5. A free particle of mass m traveling with momentum p along the z-axis scatters off the potential

$$V(\mathbf{r}) = V_0 \left[\delta^{(3)}(\mathbf{r} - \epsilon\hat{z}) - \delta^{(3)}(\mathbf{r} + \epsilon\hat{z})\right] \tag{5}$$

(a) Calculate the differential cross section $\frac{d\sigma}{d\Omega}$ in the Born approximation.

(b) Under which assumptions is this approximation valid?

6. Consider the (non-relativistic) scattering of a particle of mass m and charge e from a fixed charge distribution $\rho(\mathbf{r})$. Assume that the charge distribution is neutral, i.e., $\int d^3r\rho(\mathbf{r}) = 0$, and that is spherically symmetric, $\rho(\mathbf{r}) = \rho(r)$. Define the second moment of the distribution as

$$A = \int d^3r r^2 \rho(r). \tag{6}$$

(a) Use the Born approximation to derive the differential cross section

(b) Derive the expression for forward scattering, $\theta = 0$.

(c) Assume that $\rho(r)$ is for a neutral hydrogen atom in its ground state. Calculate A, assuming that the nucleus does not recoil.

7. Consider the (non-relativistic) scattering of an electron of mass m and momentum k through an angle θ. Calculate the differential cross-section $\frac{d\sigma}{d\Omega}$ in the Born approximation for the spin-dependent potential

$$V = e^{-\mu r^2}[A + B\vec{\sigma} \cdot \mathbf{r}], \tag{7}$$

where $\vec{\sigma}$ are the Pauli matrices and μ, A, B are constants. Assume that the initial spin is polarized along the incident direction and sum over all final spins.