1. For the *non-local separable* potential

\[ \langle r'|V|r \rangle = \lambda v(r)v(r'), \]  

(1)

with \( v \) vanishing at large radius, determine the integral equation of \( \Psi^+(r) \) and solve as much as possible. Discuss the Born series for this potential and obtain the scattering amplitude in the first Born approximation.

2. Consider the 3D repulsive potential

\[ V(r) = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases} \]  

(2)

with \( V_0 > 0 \). A particle with energy \( E = \hbar^2k^2/2m < V_0 \) is incident upon the potential.

(a) Derive the phase shift for the s-wave.

(b) How does the phase shift behave for \( V_0 \to \infty \)?

(c) Derive the total cross section in the limit of very low energy

3. Consider a particle of mass \( m \) which scatters in 3D from a potential which is a shell at radius \( a \):

\[ V(r) = -C\delta(r-a). \]  

(3)

Derive the s-wave expression for the scattering cross section in the limit of very low energy.

4. Prove

\[ \sigma_{tot} \simeq \frac{m^2}{\pi \hbar^4} \int d^3x \int d^3x' V(r)V(r') \frac{\sin^2 k|x-x'|}{k^2|x-x'|^2} \]  

(4)
in each of the following ways.

(a) By integrating the differential cross section computed using the first-order Born approximation.
(b) By applying the optical theorem to the forward-scattering amplitude in the second-order Born approximation. [Note that \( f(0) \) is real if the first-order Born approximation is used.]

5. A free particle of mass \( m \) traveling with momentum \( p \) along the \( z \)-axis scatters off the potential
\[
V(r) = V_0 \left[ \delta^{(3)}(r - \epsilon \hat{z}) - \delta^{(3)}(r + \epsilon \hat{z}) \right]
\]  
(5)

(a) Calculate the differential cross section \( \frac{d\sigma}{d\Omega} \) in the Born approximation.

(b) Under which assumptions is this approximation valid?

6. Consider the (non-relativistic) scattering of a particle of mass \( m \) and charge \( e \) from a fixed charge distribution \( \rho(r) \). Assume that the charge distribution is neutral, i.e., \( \int d^3r \rho(r) = 0 \), and that is spherically symmetric, \( \rho(r) = \rho(r) \). Define the second moment of the distribution as
\[
A = \int d^3r r^2 \rho(r).
\]  
(6)

(a) Use the Born approximation to derive the differential cross section

(b) Derive the expression for forward scattering, \( \theta = 0 \).

(c) Assume that \( \rho(r) \) is for a neutral hydrogen atom in its ground state. Calculate \( A \), assuming that the nucleus does not recoil.

7. Consider the (non-relativistic) scattering of an electron of mass \( m \) and momentum \( k \) through an angle \( \theta \). Calculate the differential cross-section \( \frac{d\sigma}{d\Omega} \) in the Born approximation for the spin-dependent potential
\[
V = e^{-\mu r^2} [A + B \vec{\sigma} \cdot \vec{r}],
\]  
(7)

where \( \vec{\sigma} \) are the Pauli matrices and \( \mu, A, B \) are constants. Assume that the initial spin is polarized along the incident direction and sum over all final spins.