

# Homework Set 5 — Scattering 2

Due May 29

1. For the *non-local separable* potential

$$\langle \mathbf{r}' | V | \mathbf{r} \rangle = \lambda v(\mathbf{r})v(\mathbf{r}'), \quad (1)$$

with  $v$  vanishing at large radius, determine the integral equation of  $\Psi^+(\mathbf{r})$  and solve as much as possible. Discuss the Born series for this potential and obtain the scattering amplitude in the first Born approximation.

2. Consider the 3D repulsive potential

$$V(r) = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases} \quad (2)$$

with  $V_0 > 0$ . A particle with energy  $E = \hbar^2 k^2 / 2m < V_0$  is incident upon the potential.

- (a) Derive the phase shift for the  $s$ -wave.
  - (b) How does the phase shift behave for  $V_0 \rightarrow \infty$ ?
  - (c) Derive the total cross section in the limit of very low energy
3. Consider a particle of mass  $m$  which scatters in 3D from a potential which is a shell at radius  $a$ :

$$V(r) = -C\delta(r - a). \quad (3)$$

Derive the  $s$ -wave expression for the scattering cross section in the limit of very low energy.

4. Prove

$$\sigma_{\text{tot}} \simeq \frac{m^2}{\pi \hbar^4} \int d^3x \int d^3x' V(r)V(r') \frac{\sin^2 k|\mathbf{x} - \mathbf{x}'|}{k^2|\mathbf{x} - \mathbf{x}'|^2} \quad (4)$$

in each of the following ways.

- (a) By integrating the differential cross section computed using the first-order Born approximation.

(b) By applying the optical theorem to the forward-scattering amplitude in the *second-order* Born approximation. [Note that  $f(0)$  is real if the first-order Born approximation is used.]

5. A free particle of mass  $m$  traveling with momentum  $p$  along the  $z$ -axis scatters off the potential

$$V(\mathbf{r}) = V_0 [\delta^{(3)}(\mathbf{r} - \epsilon\hat{z}) - \delta^{(3)}(\mathbf{r} + \epsilon\hat{z})] \quad (5)$$

(a) Calculate the differential cross section  $\frac{d\sigma}{d\Omega}$  in the Born approximation.

(b) Under which assumptions is this approximation valid?

6. Consider the (non-relativistic) scattering of a particle of mass  $m$  and charge  $e$  from a fixed charge distribution  $\rho(\mathbf{r})$ . Assume that the charge distribution is neutral, i.e.,  $\int d^3r \rho(\mathbf{r}) = 0$ , and that it is spherically symmetric,  $\rho(\mathbf{r}) = \rho(r)$ . Define the second moment of the distribution as

$$A = \int d^3r r^2 \rho(r). \quad (6)$$

(a) Use the Born approximation to derive the differential cross section

(b) Derive the expression for forward scattering,  $\theta = 0$ .

(c) Assume that  $\rho(r)$  is for a neutral hydrogen atom in its ground state. Calculate  $A$ , assuming that the nucleus does not recoil.

7. Consider the (non-relativistic) scattering of an electron of mass  $m$  and momentum  $k$  through an angle  $\theta$ . Calculate the differential cross-section  $\frac{d\sigma}{d\Omega}$  in the Born approximation for the spin-dependent potential

$$V = e^{-\mu r^2} [A + B \vec{\sigma} \cdot \mathbf{r}], \quad (7)$$

where  $\vec{\sigma}$  are the Pauli matrices and  $\mu, A, B$  are constants. Assume that the initial spin is polarized along the incident direction and sum over all final spins.