Homework Set 7

Due October 15, 2020

1. Show that the 3×3 matrices $G_i (i = 1, 2, 3)$ whose elements are given by

$$(G_i)_{jk} = -i\hbar\epsilon_{ijk}, \tag{1}$$

where j and k are the row and column indices, satisfy the angular-momentum commutation relations.

What is the physical (or geometric) significance of the transformation matrix that connects G_i to the more usual 3×3 representations of the angular-momentum operator J_i with J_3 taken to be diagonal?

Relate your result to

$$\mathbf{V} \to \mathbf{V} + \hat{\mathbf{n}}\delta\phi \times \mathbf{V} \tag{2}$$

under infinitesimal rotations. (*Note*: This problem may be helpful in understanding the photon spin.)

2. (a) Let $\mathbf{J}$ be an angular momentum operator. Using the fact that J_x, J_y, J_z satisfy the usual angular momentum commutation relations, and

$$J_\pm \equiv J_x \pm iJ_y, \tag{3}$$

prove

$$J^2 = J_z^2 + J_+J_- - \hbar J_z. \tag{4}$$

(b) Using this, derive the coefficient c_- defined by

$$J_-|jm\rangle = c_-|j, m-1\rangle \tag{5}$$

3. Consider an orbital angular-momentum eigenstate $|l = 2, m = 0\rangle$. Suppose this state is rotated by an angle β about the y-axis. Find the probability for the new state to be found in m=0, ±1, and ±2.

4. A spin-1 particle has the component of its spin in the direction

$$\hat{n} = \frac{1}{\sqrt{3}}(1, 1, 1), \tag{6}$$

measured, and the result is $\hbar$. Subsequently S_z is measured, with various probabilities of the three possible outcomes.

Let R be a rotation that maps the $\hat{z}$ axis into $\hat{n}$,

$$R\hat{z} = \hat{n}. \tag{7}$$

Express the probabilities of the three possible measurement outcomes in terms of the rotation matrix

$$D^1_{mm'}(R) \equiv \langle j = 1, m'|U(R)|j = 1, m\rangle \tag{8}$$

Work out this matrix to find the probabilities explicitly. (You may use tables of d-matrices).