Homework Set 3

Due September 10, 2020

1. The projection postulate of quantum mechanics says that if a system is described by a pure state $|\psi\rangle$ (here assumed to be normalized), then after a measurement of the operator A producing eigenvalue a_n, the system is described by the (normalized) state

$$|\psi'\rangle = \frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}, \tag{1}$$

   where P_n is the projector onto the n-th eigenspace of A.

   I.e., for no degeneracy

$$P_n = |n\rangle\langle n|. \tag{2}$$

   When there is a degeneracy of order M, we define an orthonormal basis within the eigenspace, labeled by j, and

$$P_n = \sum_{j_1}^{M} |nj\rangle\langle nj| \tag{3}$$

   Suppose instead the system is a mixed state described by a density operator ρ (assumed normalized). What is the (normalized) density operator ρ' after a measurement with result a_n? Express your answer in terms of the original density operator ρ. Do not assume the eigenvalue a_n is nondegenerate.

2. Find the linear combination of eigenkets of the S_z operator, $|+\rangle$ and $|-\rangle$, that maximize the uncertainty in $\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle$. Show that for the combination you find that the uncertainty relation is not violated.

3. A three-dimensional ket space is spanned by an orthonormal basis $|1\rangle$, $|2\rangle$, and $|3\rangle$. In this basis, the operators A and B are represented by

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}, \quad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}. \tag{4}$$

   (a) Do A or B have degenerate eigenvalues?

   (b) Show that A and B commute.

   (c) Find a new set of kets that are the simultaneous eigenkets of A and B. Specify the eigenvalues of A and B for each ket. Are these eigenvalues sufficient to uniquely specify each ket?

4. Let $f(\mathbf{x})$ be an arbitrary function of $\mathbf{x}$ and $g(\mathbf{p})$ an arbitrary function of $\mathbf{p}$. Evaluate the commutators, $[p_i, f(\mathbf{x})]$ and $[x_i, g(\mathbf{p})]$