

# Homework Set 11

Due November 23, 2020

1. Consider a spinless particle in a two-dimensional infinite square well

$$V = \begin{cases} 0, & \text{for } 0 \leq x \leq a, 0 \leq y \leq a \\ \infty, & \text{otherwise} \end{cases} \quad (1)$$

- (a) Find the energy eigenstates and the corresponding energy spectrum.
- (b) What are the energy eigenvalues for the three lowest states? Is there any degeneracy?
- (c) Now add a weak potential

$$V_1 = \lambda xy, \quad 0 \leq x \leq a, 0 \leq y \leq a \quad (2)$$

Treating this as a weak perturbation, answer the following

- i. Is the energy shift due to the perturbation linear or quadratic in  $\lambda$  for each of the three states?
  - ii. Obtain expressions for the energy shifts of the three lowest states accurate to order  $\lambda$ .
  - iii. Draw an energy diagram with and without the perturbation for the three energy states. Make sure to specify which unperturbed state is connected to which perturbed state.
2. Consider a one-dimensional simple harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}. \quad (3)$$

Estimate the ground state energy by using

$$\psi_0 = e^{-\beta|x|} \quad (4)$$

as a trial wave function with  $\beta$  a parameter to be varied.

You may use

$$\int_0^{\infty} e^{-\alpha x} x^n dx = \frac{n!}{\alpha^{n+1}} \quad (5)$$