Homework Set 10

Due November 6, 2020

1. (a) Construct a spherical tensor of rank 1 out of two different vectors $\mathbf{U} = (U_x, U_y, U_z)$ and $\mathbf{V} = (V_x, V_y, V_z)$.

   Explicitly write $T^{(1)}_{\pm 1,0}$ in terms of $U_{x,y,z}$ and $V_{x,y,z}$.

   (b) Construct a spherical tensor of rank 2 out of two different vectors $\mathbf{U}$ and $\mathbf{V}$.

   Write down explicitly $T^{(2)}_{\pm 2,\pm 1,0}$ in terms of $U_{x,y,z}$ and $V_{x,y,z}$.

2. Consider a spinless particle bound to a fixed center by a central force potential.

   (a) Relate, as much as possible, the matrix elements

$$\langle n', l', m'| \mp \frac{1}{\sqrt{2}}(x \pm iy)|n, l, m\rangle \tag{1}$$

   and

$$\langle n', l', m'|z|n, l, m\rangle \tag{2}$$

   using *only* the Wigner-Eckart theorem. Make sure to state under what conditions the matrix elements are nonvanishing.

   (b) Do the same problem using wave functions

$$\psi(\mathbf{x}) = R_{nl}(r)Y_l^m(\theta, \phi) \tag{3}$$

3. (a) Write xy, xz, and $(x^2 - y^2)$ as components of a spherical (irreducible) tensor of rank 2.

   (b) The expectation value

$$Q \equiv e\langle \alpha, j, m = j|(3z^2 - r^2)|\alpha, j, m = j\rangle \tag{4}$$

   is known as the *quadrupole moment.* Evaluate

$$e\langle \alpha, j, m'|(x^2 - y^2)|\alpha, j, m = j\rangle \tag{5}$$

where $m' = j, j-1, j-2, \ldots$, in terms of Q and appropriate Clebsch-Gordan coefficients