Homework Set 6

Due October 2

1. Find the eigenvalues and eigenvectors of

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{1}$$

   Suppose an electron is in the spin state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. If S_y is measured, what is the probability of the result $+\hbar/2$, in terms of α and β?

2. In the 2-dimensional space of a spin-1/2 particle, consider a sequence of Euler rotations represented by

$$U(\alpha, \beta, \gamma) = \exp\left(\frac{-i\sigma_3\alpha}{2}\right) \exp\left(\frac{-i\sigma_2\beta}{2}\right) \exp\left(\frac{-i\sigma_3\gamma}{2}\right) \tag{2}$$

$$= \begin{pmatrix} e^{-i(\alpha+\gamma)/2}\cos\frac{\beta}{2} & -e^{-i(\alpha-\gamma)/2}\sin\frac{\beta}{2} \\ e^{i(\alpha-\gamma)/2}\sin\frac{\beta}{2} & e^{i(\alpha+\gamma)/2}\cos\frac{\beta}{2} \end{pmatrix}. \tag{3}$$

   Because of the group properties of rotations, we expect that this sequence of operations is equivalent to a *single* rotation about some axis by an angle θ. Find θ.

3. (a) Consider a pure ensemble of identically prepared spin-1/2 systems. Suppose the expectation values $\langle S_x \rangle$ and $\langle S_z \rangle$ are known, as well as the sign of $\langle S_y \rangle$.

   Show how we may determine the state vector. Why is it unnecessary to know the magnitude of $\langle S_y \rangle$?

   (b) Consider a mixed ensemble of spin-1/2 systems. Suppose the ensemble averages $[S_x]$, $[S_y]$, and $[S_z]$ are all known. Show how we may construct the 2×2 density matrix that characterizes the ensemble.