

Homework Set 4

Due September 4

1. In this problem we consider the spreading of wave packets of a free particle in one dimension. The Hamiltonian is

$$H = \frac{p^2}{2m} \quad (1)$$

- (a) Let the wave function at time $t = 0$ be

$$\psi_0(x) = \psi(x, t = 0) = Ce^{-x^2/(4L^2)}, \quad (2)$$

where C and L are constants (with L real).

Normalize the wave function and find C , choosing C real and positive, $C > 0$.

Then determine $\langle x \rangle$, $\langle p \rangle$ and $\Delta x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ at $t = 0$.

- (b) Compute the initial momentum space wave function $\psi_0(p)$, and from that compute $\Delta p \equiv \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

Compare $\Delta x \Delta p$ to the minimum value allowed by the uncertainty principle

- (c) Now compute $\psi(p, t)$ for any time t . Use it to find $\langle p \rangle$ and Δp for any time t .

- (d) Now compute $\psi(x, t)$ for any time t . Express your answer in terms of the quantity,

$$D = L + \frac{i\hbar t}{2mL}, \quad (3)$$

(a useful abbreviation). Then find $\langle x \rangle$ and Δx for any time t . Notice what happens to the product $\Delta x \Delta p$ for times $t > 0$.

- (e) If you had not done a detailed calculation you could estimate Δx as a function of t by using the uncertainty principle. Do this and compare to the results of the detailed calculation

2. Consider a single, spinless particle moving in one dimension, confined by rigid walls at $x = 0$ and $x = a$.

That is, a particle in an infinite square well

$$H = \frac{p^2}{2m} + V(x), \quad (4)$$

$$V(x) = \begin{cases} \infty & x > a \\ 0 & 0 \leq x \leq a, \\ \infty & x < 0 \end{cases} \quad (5)$$

so that the wavefunction satisfies the free particle Schrödinger equation for $0 \leq x \leq a$, but the probability of measuring the particle with position $x < 0$ or $x > a$ is zero.

- (a) Find an expression for the energy eigenstates $\psi_n(x)$ and eigenvalues E_n . Is there any degeneracy?
- (b) For each energy eigenstate, evaluate

$$\Delta x^2 \Delta p^2 \equiv (\langle x^2 \rangle - \langle x \rangle^2) (\langle p^2 \rangle - \langle p \rangle^2) \quad (6)$$

- (c) For which energy eigenstate is this the smallest? How does it compare to the minimum allowed by the uncertainty principle?